Unified models of the cosmological dark sector

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W.S. Hipólito-Ricaldi, H.E.S. Velten and W.Z., JCAP **06** (2009) 016; arXiv:0902.4710
W.S. Hipólito-Ricaldi, H.E.S. Velten and W.Z., PRD **82**, 063507 (2010); arXiv:1007.0675

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The cosmic substratum

PREVAILING VIEW:

Two dynamically dominating components:

Dark Matter (DM), energy density ρ_M, pressure $p_M \ll \rho_M$

Dark Energy (DE), energy density ρ_X, pressure $p_X = w\rho_X$

Equation of state parameter w ?

DE candidates

- **Cosmological constant** $\Leftrightarrow$ **Vacuum energy**
- **Scalar fields**
- **Holographic dark energy**
- **Alternative theories**

Dark sector

Both DM and DE manifest themselves only gravitationally

Separate substances?

Different manifestations of one single component?

Unified description:

- **(Generalized) Chaplygin gas**
- **BULK VISCOUS FLUID**

Chaplygin gas

Prototype of a unified dark-sector model
Equation of state

$$p = -\frac{A}{\rho}$$

(A. Yu. Kamenshchik, U. Moschella and V. Pasquier, Phys.Lett. **B511**, 265 (2001))

Homogeneous and isotropic Universe
Energy density:

$$\rho = \left[A + Ba^{-3/2}\right]^{1/2}$$

$a \ll 1 \quad \Rightarrow \quad \rho \propto a^{-3}$ **matter**

$a \gg 1 \quad \Rightarrow \quad \rho \propto A =$ **constant**

Generalized Chaplygin gas

Phenomenological generalization

$$p = -\frac{A}{\rho^{\alpha}}$$

(M. C. Bento, O. Bertolami and A. A. Sen, Phys. Rev. **D66**, 043507 (2002)

$$\rho = \left[A + Ba^{-3(1+\alpha}\right]^{1/(1+\alpha)}$$

$a \ll 1 \quad \Rightarrow \quad \rho \propto a^{-3}$

$a \gg 1 \quad \Rightarrow \quad \rho \propto A = \text{constant}$

Background evolution well described

(R. Colistete Jr. and J.C. Fabris, CQG 22, 2813 (2005))

The end of unified dark matter?

H.B. Sandvik, M. Tegmark, M. Zaldariaga and I. Waga, Phys. Rev. D 69, 123524 (2004)

Problem: finite adiabatic sound speed

$$\frac{\dot{p}}{\dot{\rho}} = -\alpha \frac{p}{\rho}$$

$$\frac{p}{\rho} < 0 \quad \Rightarrow \quad \frac{\dot{p}}{\dot{\rho}} > 0$$

$$\hat{p} = \frac{\dot{p}}{\dot{\rho}} \hat{\rho}$$

Oscillations in the matter-power spectrum

Generalized Chaplygin gas

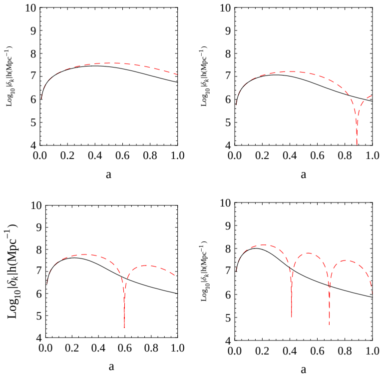


Figure: **Density fluctuations as function of the scale factor a for $\alpha = 1/2$ and $q_0 = -0.5$. k = 0.5 (top left), k = 0.7 (top right), k = 1 (bottom left) and k = 1.5 (bottom right), all in units of $hMpc^{-1}$ (dashed curves). Solid curves: bulk viscous model**

Way out?

Nonadiabatic perturbations?

(R.R.R. Reis, I. Waga, M.O. Calvão e S.E. Joràs, Phys. Rev. **D68**, 061302 (2003)
L. Amendola, I. Waga and F. Finelli, JCAP **0511**, 009 (2005).)

Assume existence of nonadiabatic pressure perturbations:

$$\hat{p} = \hat{p}_{ad} + \hat{p}_{nad} \approx 0$$

Cancelation mechanism introduced **ad hoc**

"Silent quartessence"

Physical motivation?

Bulk viscous fluid

Spatial homogeneity and isotropy:

No heat flux, no anisotropic pressure

Only dissipative phenomenon: scalar bulk viscous pressure

Energy momentum tensor

$$T^{ik} = \rho\, u^i u^k + (p + \Pi)\ h^{ik}$$

Energy balance

$$\dot{\rho} = -\Theta\ (\rho + p + \Pi)$$

Properties of Π?

Bulk viscosity coefficient

Linear relation

$$\mathbf{\Pi} = -\zeta\mathbf{\Theta} \quad \Rightarrow \quad \mathbf{\Pi} = -\mathbf{3H}\zeta$$

$\zeta > 0$ – **coefficient of bulk viscosity**

Negative pressure contribution

$$T^{ik} = \rho\, u^i u^k + (p + \Pi)\ h^{ik}$$

Origin of Π?

Bulk viscous pressure

Deviations from equilibrium: Cosmic bulk viscosity
W. Israel, JMP 4 (1963) 1163; S. Weinberg, ApJ. 168 (1971) 175

Physical origin: **different cooling rates**
W.Z. MNRAS **280** 1239 (1996); W.Z. MNRAS **288** 665 (1997)

Radiation: $T \propto a^{-1}$ Matter: $T \propto a^{-2}$

"Standard" interactions: $\Pi < p$

To obtain $\Pi \lesssim \rho$: **Non-standard interactions** required

(W.Z, D.J. Schwarz, A.B. Balakin, and D. Pavón, Phys. Rev. D **64**, 063501 (2001))

Unified Model of dark matter and dark energy

$$p \ll \rho \ , \quad \Pi = -\Theta\zeta \ , \quad \zeta \propto \rho^{\nu}$$

Background dynamics

$$\rho = \left[A + B \left(\frac{a_0}{a} \right)^{\frac{3}{2}(1-2\nu)} \right]^{\frac{2}{1-2\nu}}$$

Coincides with dynamics of a generalized Chaplygin gas

$\mathbf{a} \ll \mathbf{a_0}$ $\quad \rho \propto a^{-3}$ **dark matter**

$\mathbf{a} \gg \mathbf{a_0}$ $\quad \rho \propto \mathrm{const}$ **dark energy**

Correspondence

Bulk viscous fluid

$$\Pi = -A\rho^{\nu+1/2}$$

Compare **generalized Chaplygin gas**

$$p = -\frac{A}{\rho^{\alpha}}$$

Correspondence: $\alpha = -\left(\nu + \frac{1}{2}\right)$

Background dynamics is equivalent
(M. Szydłowski and O. Hrycyna, AnnalsPhys.**322**, 2745 (2007);
R. Colistete Jr., J.C. Fabris, J. Tossa and W.Z., Phys. Rev. **D76**, 103516 (2007))

Background dynamics in terms of q_0

Present value of the deceleration parameter $\mathbf{q_0}$

Background energy density

$$\frac{\rho}{\rho_0} = \left(\frac{1}{9}\right)^{\frac{1}{1-2\nu}} \left[1 - 2q_0 + 2\left(1 + q_0\right) a^{-\frac{3}{2}(1-2\nu)}\right]^{\frac{2}{1-2\nu}}$$

Correspondence Chaplygin gas: $\alpha = -\left(\nu + \frac{1}{2}\right)$

Viscosity-induced late-time accelerated expansion:
T. Padmanabhan and S. M. Chitre, Phys. Lett. A **120**, 433 (**1987**)!

Nonadiabaticity

Pressure perturbations

$$\Pi \propto -\rho^{\nu}\Theta \quad \Rightarrow \quad \hat{\Pi} = \left[\frac{\hat{\Theta}}{\Theta} + \nu\frac{\hat{\rho}}{\rho}\right]\Pi$$

Nonadiabaticity:

$$\hat{\Pi} - \frac{\dot{\Pi}}{\dot{\rho}}\hat{\rho} = \Pi\left(\frac{\hat{\Theta}}{\Theta} - \frac{1}{2}\frac{\hat{\rho}}{\rho}\right) \neq 0$$

Perturbations of the expansion scalar: Raychaudhuri equation

$$\dot{\Theta} + \frac{1}{3}\Theta^2 + 2\left(\sigma^2 - \omega^2\right) - \dot{u}^a_{;a} + 4\pi\, G\, \left(\rho + 3\Pi\right) = 0$$

"Nonlocal" equation of state

Pressure perturbations are determined by perturbations of the energy density AND by perturbation of the expansion scalar

This amounts to (first order)

$$\hat{\boldsymbol{\Pi}} = \hat{\boldsymbol{\Pi}}(\hat{\rho}, \dot{\hat{\rho}})$$

$\Pi = \Pi(\rho)$ **only in the background**

"Non-local" dependence

(W.Z. IJMPD **17**, 651 (2008) (arXiv:0705.2131))

Perturbation dynamics

Fractional density contrast δ

$$\delta'' + f\,(a)\,\delta' + g\,(a)\;\delta = 0$$

with

$$f\,(a) = \frac{1}{a}\left[\frac{3}{2} - 6\frac{\Pi}{\rho} + 3\nu\frac{\Pi}{\rho} - \frac{1}{3}\frac{\Pi}{\rho+\Pi}\,\frac{k^2}{H^2a^2}\right]$$

and

$$g\,(a) = -\frac{1}{a^2}\left[\frac{3}{2} + \frac{15}{2}\frac{\Pi}{\rho} - \frac{9}{2}\frac{\Pi^2}{\rho^2} - 9\nu\frac{\Pi}{\rho} - \frac{\Pi}{\rho}\left(\frac{\Pi}{\rho+\Pi} + \nu\right)\frac{k^2}{H^2a^2}\right]$$

Perturbation equation Chaplygin gas

Fractional density contrast δ_c

$$\delta_c'' + f_c(a)\,\delta_c' + g_c(a)\,\delta_c = 0$$

$$f_c(a) = \frac{1}{a}\left[\frac{3}{2} - \frac{15}{2}\frac{p}{\rho} - 3\alpha\frac{p}{\rho}\right]$$

$$g_c(a) = -\frac{1}{a^2}\left[\frac{3}{2} + 12\frac{p}{\rho} - \frac{9}{2}\frac{p^2}{\rho^2} + 9\alpha\frac{p}{\rho} + \alpha\frac{p}{\rho}\frac{k^2}{H^2 a^2}\right]$$

Compare viscous fluid

$$f(a) = \frac{1}{a}\left[\frac{3}{2} - 6\frac{\Pi}{\rho} + 3\nu\frac{\Pi}{\rho} - \frac{1}{3}\frac{\Pi}{\rho+\Pi}\frac{k^2}{H^2 a^2}\right]$$

$\alpha = -(\nu + \frac{1}{2})$

Viscous model vs. generalized Chaplygin gas

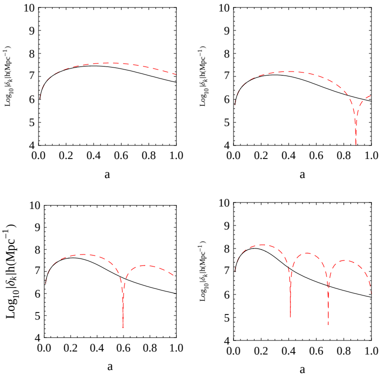


Figure: **Density fluctuations as function of the scale factor a for $\alpha = 1/2$ and $q_0 = -0.5$. k $= 0.5$ (top left), k $= 0.7$ (top right), k $= 1$ (bottom left) and k $= 1.5$ (bottom right), all in units of hMpc^{-1} (dashed curves). Solid curves: bulk viscous model**

Perturbations at high redshifts

Limiting behavior for $a \ll 1$:

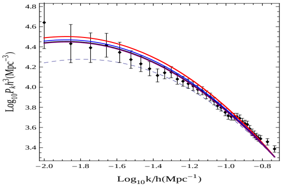
Einstein-de Sitter:

$$\delta'' + \frac{3}{2a}\,\delta' - \frac{3}{2a^2}\,\delta = 0$$

Nonadiabaticity negligible at high redshifts

Behavior indistinguishable from Chaplygin-gas and ΛCDM models

Matter power spectrum



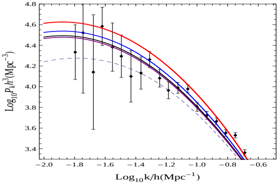


Figure: Density power spectrum for the bulk-viscous model with $\nu = -5$ (solid curves) and the ΛCDM model (dashed curves). From top to bottom the curves represent cases with $q_0 = -0.4$, $q_0 = -0.2$, $q_0 = 0$ and $q_0 = 0.1$. The curves are compared with 2dFGRS data (top) and and SDSS data (bottom).

Baryons included

Matter and viscous fluid

$$T^{ik} = T^{ik}_M + T^{ik}_V$$

Separate energy-momentum conservation

$$T^{ik}_{M\,;i} = T^{ik}_{V\,;i} = 0 \quad \Rightarrow \quad T^{ik}_{;i} = 0$$

Viscous pressure coincides with total pressure

$$p_M = 0\ , \qquad p_V = p = \Pi = -\zeta\Theta$$

Here: $\zeta =$ constant

Total two-component dynamics

The total energy density is that of a generalized Chaplygin gas:

$$\rho = \frac{\rho_0}{9}\left[1 - 2q_0 + 2\left(1 + q_0\right) a^{-\frac{3}{2}}\right]^2$$

$$\rho_M = \rho_{M0} a^{-3} \qquad \rho_V = \rho - \rho_M$$

Viscous fluid: unified model of the dark sector

Generalized Chaplygin gas ($\alpha = -1/2$):

Unified description of the unified dark sector and (baryonic) matter

Relative entropy perturbations

Definition

$$S_{MV} \equiv \frac{\hat{\rho}_M}{\rho_M} - \frac{\hat{\rho}_V}{\rho_V + p_V}$$

Inhomogeneous second-order equation

$$S''_{VM} + r(a)S'_{VM} + s(a)S_{VM} = c(a)\delta' + d(a)\delta$$

Coupling to total energy-density perturbations

$$\delta'' + f\,(a)\,\delta' + g\,(a)\;\delta = 0$$

Observed matter perturbations?

Baryonic perturbations

Combination of total and relative entropy perturbations

$$\delta_M = \frac{\rho}{\rho + p}\left[\delta - \frac{\rho_V + p}{\rho} S_{VM}\right]$$

High redshifts:

$$S_{VM} = 0 \qquad \delta_M = \delta \qquad (a \ll 1)$$

Adiabatic behavior at $a \ll 1$

Adiabatic initial conditions

Probability distribution function (PDF) for q_0

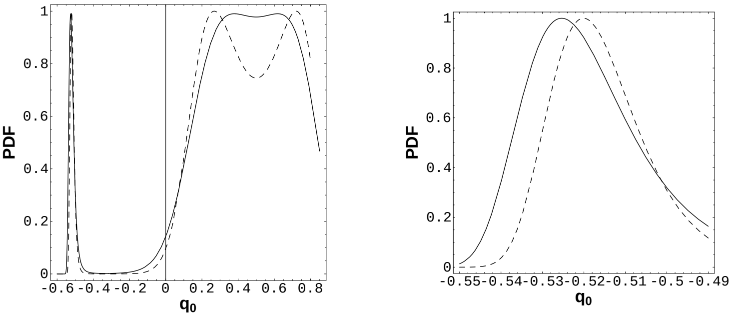


Figure: One-dimensional PDF for q_0 resulting from the 2dFGRS data (solid curve) and from the SDSS DR7 data (dashed curve). The right picture is an amplification of the peak in the region $q_0 < 0$.

Matter power spectrum

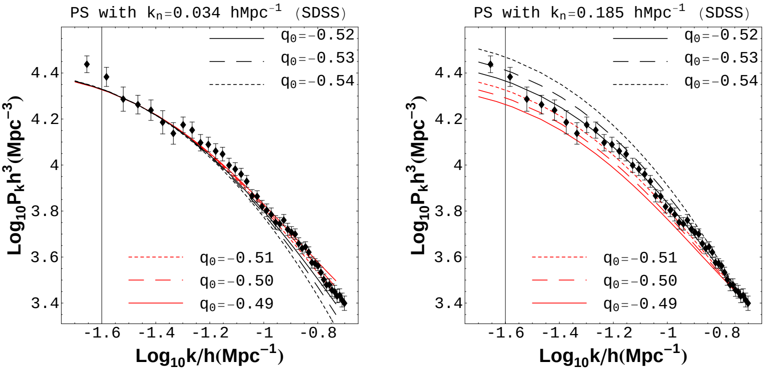


Figure: Power spectra (PS) normalized at $k_n = 0.034hMpc^{-1}$ (left panel) and at $0.185hMpc^{-1}$ (right panel) for different negative values of q_0. The PS is compared panels with the SDSS DR7 data.

Joint analysis

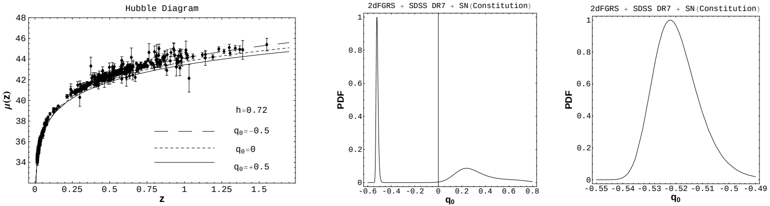


Figure: Left panel: Hubble diagram, center panel: PDF for q_0, based on a joint analysis of PS and SN data. The right panel magnifies the maximum for $q_0 < 0$.

Probability of the unified model

So far it was assumed that

$$\Omega_{M0} = \Omega_{B0} = 0.043 \qquad \Omega_{V0} = 1 - 0.043$$

Now: Ω_{M0} as **free parameter**

Most probable values?

High probability for $\Omega_{M0} = \Omega_{B0} = 0.043$**?**

Otherwise the unified model is disfavored

(**Chaplygin-gas models do not pass this test!**)

Test of the viscous unified model

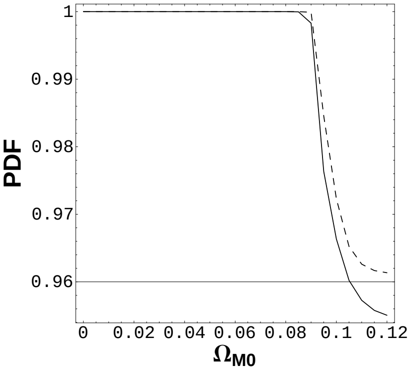


Figure: PDF for the pressureless component Ω_{M0}.

Unified Model: Generalized Chaplygin gas

- **Adiabatic (!) perturbation analysis:**
- **Generalized Chaplygin gas apparently ruled out**

  $\Rightarrow$ **Small scale oscillations and instabilities**
- **Not observed!**
- **Observed matter-power spectrum not reproduced**

Unified model: Bulk viscous fluid

- **Same background dynamics as generalized Chaplygin gas**
- **Perturbations intrinsically nonadiabatic**

  $\Rightarrow$ **No small scale oscillations and instabilities**
- **Observed matter-power spectrum reproduced**
- **"Nonadiabatic Chaplygin gas"**

Baryons and bulk viscous fluid

- **Background dynamics: generalized Chaplygin gas**
- **Total density perturbations intrinsically nonadiabatic**
- **Additionally: relative entropy perturbations**
- **Observed matter-power spectrum reproduced**
- **PDF for q_0 has a maximum at $q_0 \approx -0.5$**
- **High probability for unified dark-sector model**