QUANTUM GRAVITATIONAL FLUCTUATIONS IN DE SITTER COSMOLOGY

Spanish Relativity Meeting ERE2010

Gravity as a Crossroad in Physics

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E. Verdaguer

University of Barcelona

OUTLINE

1. Semiclassical and Stochastic Gravity
2. Large N expansion
3. Cosmological perturbations in de Sitter
4. De Sitter geometry and bitensors
5. Stress-tensor two-point correlations
6. Quantum gravitational perturbations
7. Results

SEMICLASSICAL AND STOCHASTIC GRAVITY

E. Calzetta, A. Campos, B.L. Hu, F.C. Lombardo, K. Maeda, R. Martin, A. Matacz, F.D. Mazzitelli, N.G. Phillips, A. Roura, S. Sinha,Y. Urakawa, E.V.

SEMICLASSICAL GRAVITY

- **Gravity as an effective field theory**

$$S_{grav} = \int d^4x\sqrt{-g}\left(\frac{1}{2\kappa}(R-2\Lambda)+c_1R^2+c_2C_{abcd}C^{abcd}+\frac{c_3}{M^2}R^3+\cdots\right)$$

$$\kappa \equiv 8\pi G \equiv 8\pi l_P^2 \equiv 8\pi / m_P^2$$

Range of validity below energy scale *M*.
Results from separartion of low energy from high energy gravitational quantum effects.
Terms consistent w gen covar. dimensionless coeffs
Energy expansion: higher and higher powers of derivatives of the metric.

- **Semiclassical limit**: *1/N* expansion in interaction w *N* fields keeping constant $\overline{\kappa} = \kappa N$ *(Hartle-Horowitz 81)*

SEMICLASSICAL EINSTEIN EQUATION

- Renormalization shifts coeffs up to square curvature tensors effect of high energy modes of matter (4 ren const, exp)

$$G_{ab}[g] + \Lambda g_{ab} - \alpha A_{ab}[g] - \beta B_{ab}[g] = \kappa \langle \hat{T}_{ab}[g] \rangle_{ren}$$

where

$$A^{ab} = \frac{1}{\sqrt{-g}} \frac{\delta}{\delta g_{ab}} \int d^4x \sqrt{-g}\, C_{cdef} C^{cdef}$$

$$B^{ab} = \frac{1}{\sqrt{-g}} \frac{\delta}{\delta g_{ab}} \int d^4x \sqrt{-g}\, R^2$$

$$T^{ab} = \nabla^a \phi \nabla^b \phi - \frac{1}{2} g^{ab} (\nabla^c \phi \nabla_c \phi + m^2 \phi^2) + \xi (g^{ab} \nabla^c \nabla_c - \nabla^a \nabla^b + G^{ab}) \phi^2$$

$$\hat{T}^R_{ab}[g] = \hat{T}_{ab}[g] + F_{ab}[g] \hat{I}$$ (***F*** counterterms from SDW series)

SEMICLASSICAL GRAVITY

- **Semiclassical Einstein eq**

$$G_{ab}[g] = \bar{\kappa}\langle \hat{T}_{ab}[g]\rangle_{ren}$$

$$\kappa \equiv 8\pi G \equiv 8\pi l_P^2 \equiv 8\pi / m_P^2$$

- Square curvature, higher derivative, terms on *rhs* assumed: high energy effects produce shifts in coeffs of EFT up to curvature square (renormalization)
- Klein-Gordon eq. (field op Heisenberg picture)

$$(\nabla_g^2 - m^2 - \xi R)\hat{\phi} = 0$$

- **Solutions** of semiclassical gravity:

$$\left(M, g_{ab}, \hat{\phi}, |\psi\rangle\right)$$

LIMITS OF SEMICLASSICAL GRAVITY

- Quantum fluctuations of **stress tensor** small:

If *N* (large) fields coupled to gravity
there are no fluctuations when $N \to \infty$ $\bar{G} = GN \to finite$

Next to leading order is *O(1/N)*

$$\langle \hat{T}^2 \rangle - \langle \hat{T} \rangle^2 \approx O(1/N)$$

NOISE KERNEL

- **Noise kernel** ph observable that measures **quantum flucts** of stress tensor (free of ultraviolet divergencies)

$$N_{abcd}(x,y) = \frac{1}{2}\langle 0|\{T_{ab}(x), T_{cd}(y)\}|0\rangle - \langle 0|T_{ab}(x)|0\rangle\langle 0|T_{cd}(y)|0\rangle$$

(real and +ve semidefinite). It defines a Gaussian **stochastic** tensor $\xi_{ab}[g]$

$$\langle \xi_{ab} \rangle_s = 0 \qquad \langle \xi_{ab}(x)\xi_{cd}(y)\rangle_s = N_{abcd}(x,y)$$

- **Symmetric, divergenceless,** (**traceless** for conformal field)

$$\xi_{ab}(x) = \xi_{ba}(x), \quad \nabla^a \xi_{ab}(x) = 0$$

STOCHASTIC GRAVITY

- Extend semiclassical Einstein equations to consistently account for **fluctuations** of $\hat{T}_{ab}$

- Assume linear perturbation of semiclassical solution $g_{ab} + h_{ab}$

- **Einstein-Langevin** equation: $G_{g+h} = \overline{\kappa}(\langle \hat{T} \rangle_{g+h} + \xi)$

$$G^{(1)}_{ab}[g+h] = \overline{\kappa}\langle \hat{T}^{(1)}_{ab}[g+h]\rangle_{ren} + \overline{\kappa}\xi_{ab}[g]$$

$$(\nabla^2_{g+h} - m^2 - \xi R)\hat{\phi} = 0$$

it is gauge invariant $h'_{ab} = h_{ab} + 2\nabla_{(a}\zeta_{b)}$

$$S_{ab}(g+h') = S_{ab}(g+h) + L_\zeta S_{ab}(g)$$

- E-L can be derived by functional methods: CTP Influen. func. open quantum system paradigm

SOLUTIONS OF E-L EQ & LARGE N

- E-L are stochastic equations and determine **correlations**

$$h_{ab}(x) = h^0_{ab}(x) + \bar{\kappa}\int d^4x' \sqrt{-g}\, G^{ret}_{abcd}(x,x')\xi^{cd}(x')$$

$$\langle h_{ab}(x)h_{cd}(y)\rangle_s = \langle h^0_{ab}(x)h^0_{cd}(y)\rangle_s + \bar{\kappa}^2 \iint G^{ret}_{abef}(x,x')N^{efgh}(x',y')G^{ret}_{ghcd}(y',y)$$

Intrinsic fluctuations + **Induced fluctuations**

(flucts due to initial state, act) (due to matter field flucts, pas)

- It can be shown (*Roura-EV*) that **q. metric** correl. in *1/N*:

$$\frac{1}{2}\langle\left\{\hat{h}_{ab}(x),\hat{h}_{cd}(y)\right\}\rangle = \langle h_{ab}(x)h_{cd}(y)\rangle_s$$

includes matter loops but no graviton loops.

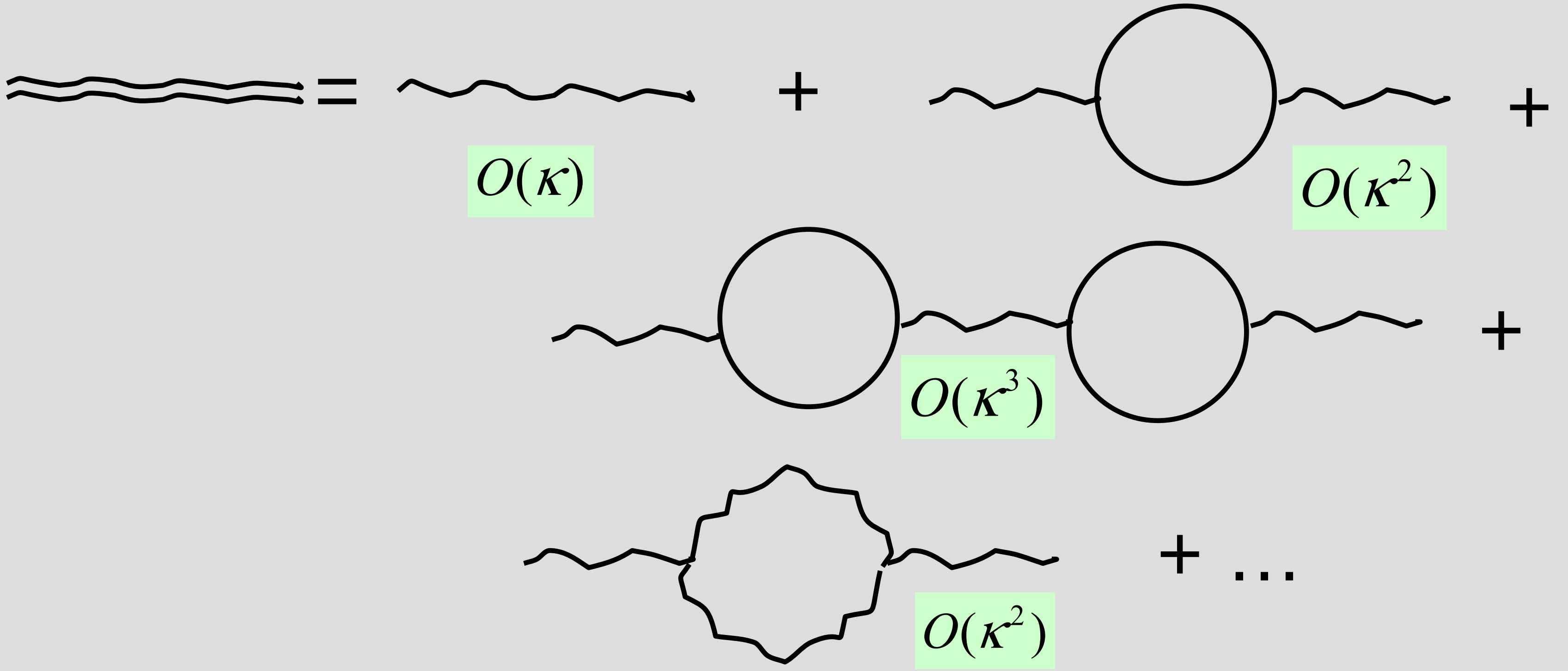
LARGE N EXPANSION

- The **large** *N* expansion goes beyond a perturbative expansion in the coupling constant
- Resums and rearranges Feynman perturbative series, including self-energies
- For gravity, graviton loops are higher order than matter loops. Zero order $1/N \to 0$: **semiclassical gravity**

LARGE N EXPANSION

- **Perturbative** Quantum Gravity

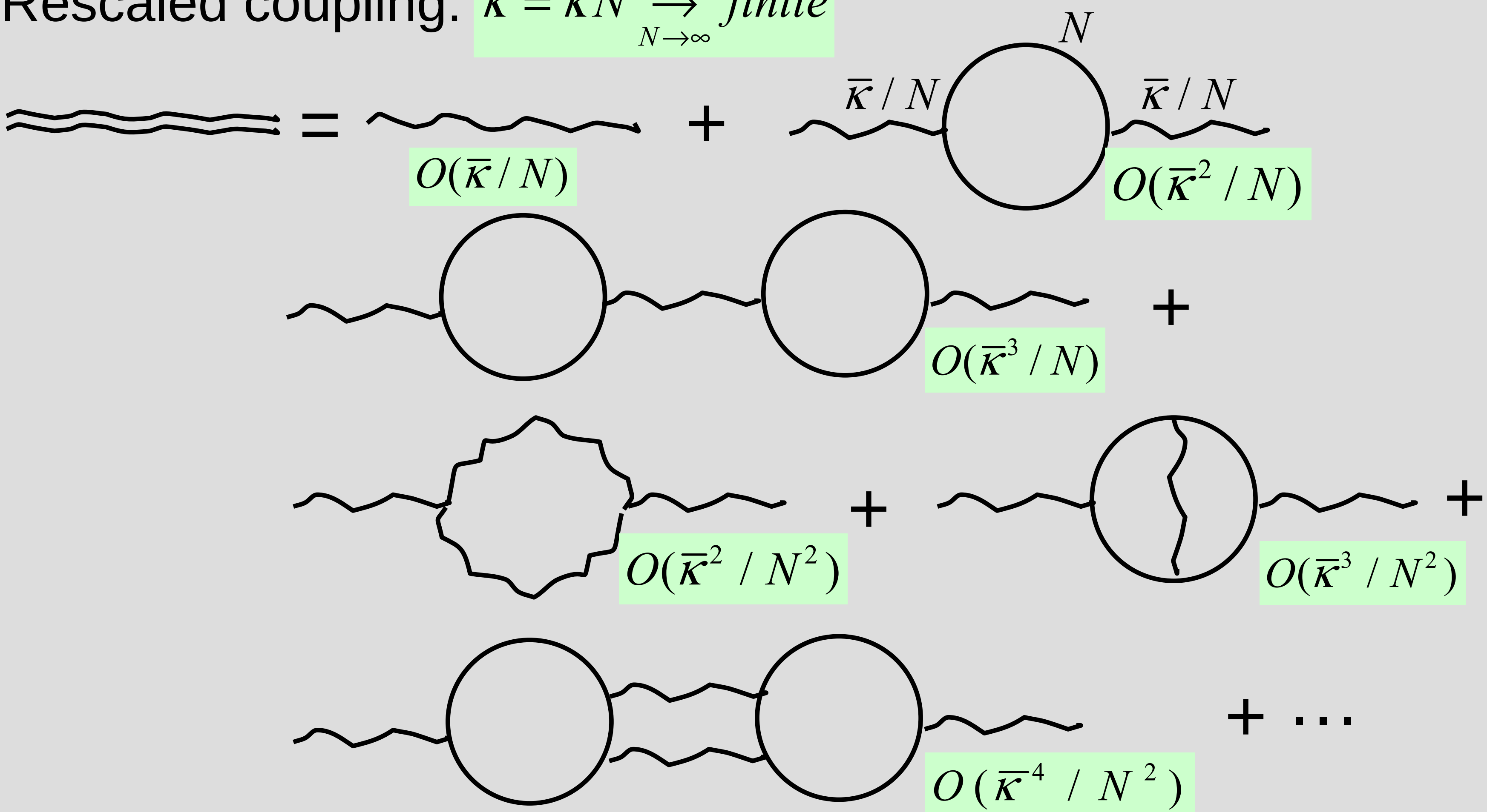
$$S = \frac{1}{2\kappa}\int d^4x\left(\partial_a h \partial^a h + h(\partial h)^2 \ldots\right) + \frac{1}{2}\int d^4x(\partial_a \phi \partial^a \phi + m^2\phi^2) + \int d^4x\left(h(\partial\phi)^2 + \ldots\right)$$



LARGE N EXPANSION

$$S = \frac{N}{2\bar{\kappa}}\int d^4x\left(\partial_a h\partial^a h + h(\partial h)^2 \ldots\right) + \frac{1}{2}\sum_j^N \int d^4x(\partial_a\phi_j\partial^a\phi_j + m^2\phi_j^{\,2}) + \sum_j^N \int d^4x\left(h(\partial\phi_j)^2 + \ldots\right)$$

Rescaled coupling: $\bar{\kappa} = \kappa N \underset{N\to\infty}{\longrightarrow} finite$



QUANTUM GRAVITATIONAL FLUCTUATIONS IN DE SITTER SPACETIME

G. Perez-Nadal, A. Roura, E.V.

COSMOLOGY & DE SITTER

- Existence of early inflationary phase successfully explains anisotropies of CMB and large scale structure.
- Present universe accelerated expansion can be driven by a cosmological constant
- Geometry of both phases is close to **de Sitter** spacetime
- Cosmological perturbations in **dS** thus extensively studied

COSMOLOGICAL PERTURBATIONS IN DE SITTER

- Standard analysis based on linearized calculation of two-point metric perturbations (tree level). Tensor pert. do not couple to matter fields (linear) (*Starobinsky 79, Mukhanov 81,92*)

- Need for loop corrections emphasized recently (test perturbat th in dS, use CTP (in-in), stochastic gravity) (*Weinberg 05, Sloth 06, Seery 07, Urakawa-Maeda 08, Adshead 09, Senatore-Zaldarriaga 09*).
  Discriminate different inflationary models with same tree level resuts (*Urakawa-Tanaka 09*)

INTERACTING FIELD THEORIES IN DS

- Plagued with IR divergencies from loop diagrams
  IR modes out-horizon undergo decohernce become classical
  Using a Fokker-Planck eq can go beyond (*Starobinsky 86, Starobinsky-Yokohama 94, Riotto-Sloth 08* also use 2PI)

- **Interacting dS invariant vacuum** for cubic and quartic self-int well behaved in IR and stable (*Marolf-Morrison 10*), but for for low-mass $m \ll H$ perturbative series do not decrease: superhorizon large fluctuations. Analogies in condensed matter near a critical point (*Burgess at al 10*)

- Interacting scalar fields are unstable? (*Polyakov 08,10*)
  But "in-out" propagators are IR divergent. Use "in-in" propag instabilties not present (*Higuchi 09, Alvarez-Vidal 09,10*)

GRAVITATIONAL PERTURBATIONS IN DE SITTER

- Generally assumed **dS** backd not affected by large q corrects.

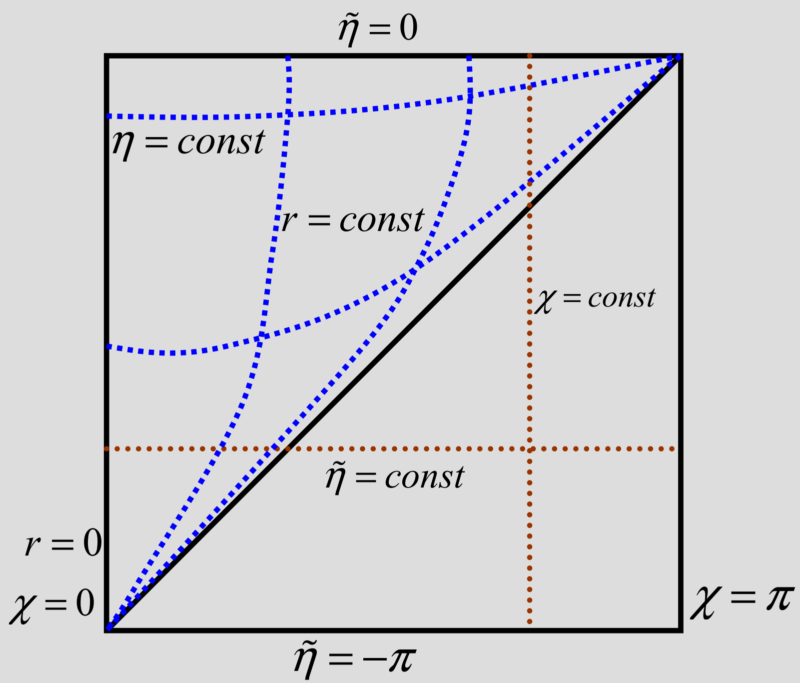
- There are claims that loop corrections could give significant backreaction effects (*Abramo 97,Losic-Unruh 05*) Nonlinear effects dom by infrared modes significant: may break dS invariance, screen cc (*Tsamis-Woodard 96,05,* but gauge dependence *Garriga-Tanaka 08*)

- We study q metric perturbations around dS including matter loops and keeping **dS** invariance (*Perez-Nadal,Roura,EV 10*) in framework of large N app. Aim: find two-point f to *O(1/N)* Consider free field interacting with gravity

DE SITTER CONFORMAL DIAGRAM

We get $ds^2 = \frac{1}{H^2 \sin^2 \tilde{\eta}}\left(-d\tilde{\eta}^2 + d\chi^2 + \sin^2 \chi (d\theta^2 + \sin^2 \theta d\varphi^2)\right) \quad -\pi < \tilde{\eta} < 0, \quad 0 \le \chi < \pi$

covers entire dS. Closed univ. contracts for $-\pi < \tilde{\eta} < -\pi/2$ expands for $-\pi/2 < \tilde{\eta} < 0$ flat, closed, open dS describe same spacetime in diff coord. Any hypersurface has constant energy density (unlike other FRW).



Conformal diagram of dS

DE SITTER GEOMETRY

- dS is an hyperboloid in (*D+1*)-Minkowski defined as

$$\eta_{AB} X^A(x) X^B(x) = H^{-2} = 1, \quad A, B = 0, \ldots, D$$

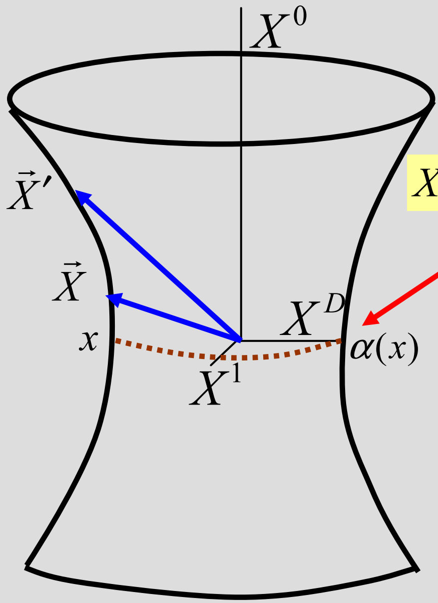
- define biscalar: $Z(x,x') = \eta_{AB} X^A(x) X^B(x')$

  invarinat under the dS group (**isometry** σ) $X^A(\sigma(x)) = \Lambda^A_{\ B} X^B(x)$

  Lorentz t

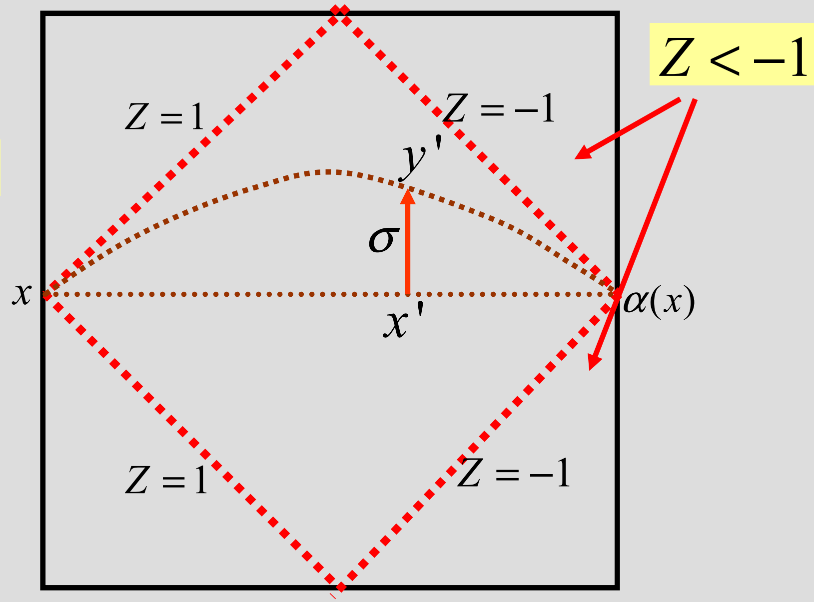
- define Mink. distance $d^2(x,x') = \eta_{AB}\left(X^A(x) - X^A(x')\right)\left(X^B(x) - X^B(x')\right)$

$$d^2 = 2(1-Z)$$

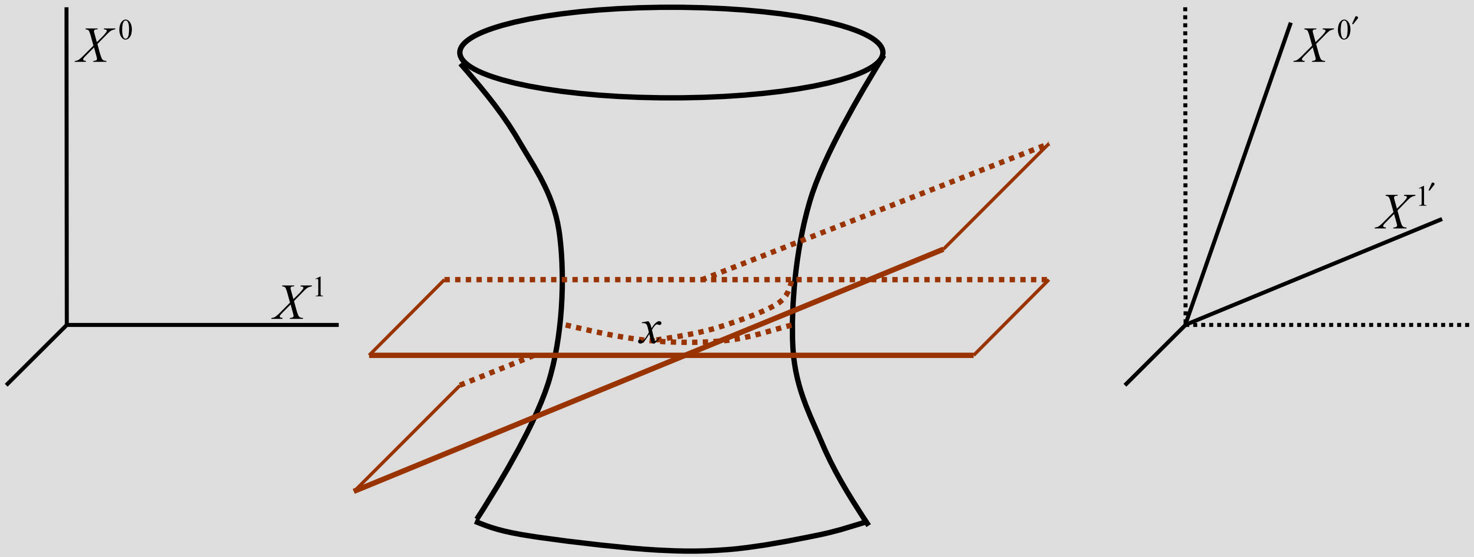


$X^A(\alpha(x)) = -X^A(x)$

antipodal



LORENTZ BOOST



DE SITTER BITENSORS

• physical distance in spatially flat coordinates is also:

$$d^2(x,x') = e^{2Ht}\delta_{ij}\left(x^i(x) - x^i(x')\right)\left(x^j(x) - x^j(x')\right)$$

• **geodesic distance** *(x,x')*:

$$\mu(x,x') = \int_0^1 d\lambda \left(g_{ab}(x(\lambda))v^a(\lambda)v^b(\lambda)\right)^{1/2}$$

• *Allen-Jacobson (86):* Any **maximamally symmetric** bitensor is a linear combination of products of:

$$n_a(x,x') = \nabla_a \mu, \quad n_{a'}(x,x') = \nabla_{a'}\mu, \quad g_{ab'}(x,x'), \quad g_{ab}(x,x'), \quad g_{a'b'}(x,x')$$

parallel-transports a vector from *x'* to *x* along the geodesic

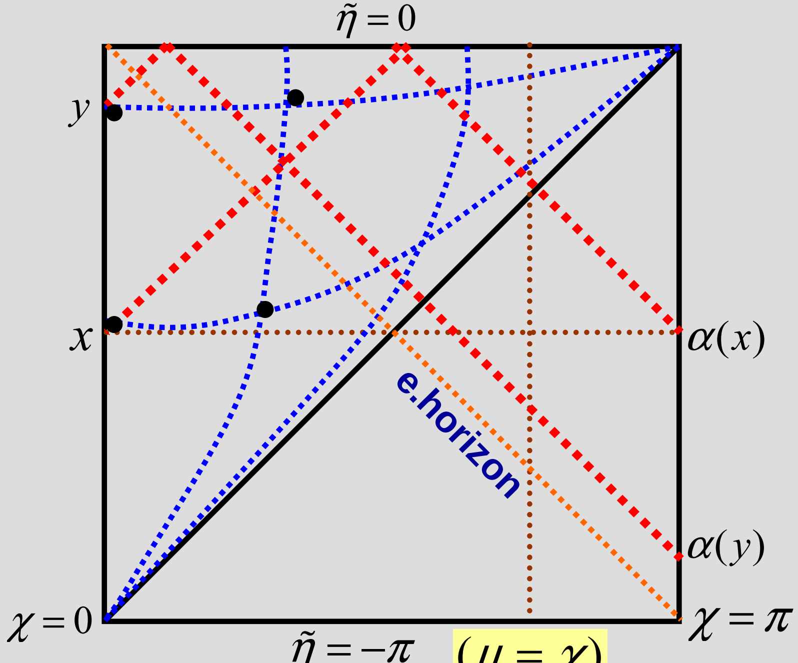
DE SITTER GEOMETRY

- covariant derivatives of basic bitensors in dS (th) *(Allen-Jacobson 86)*

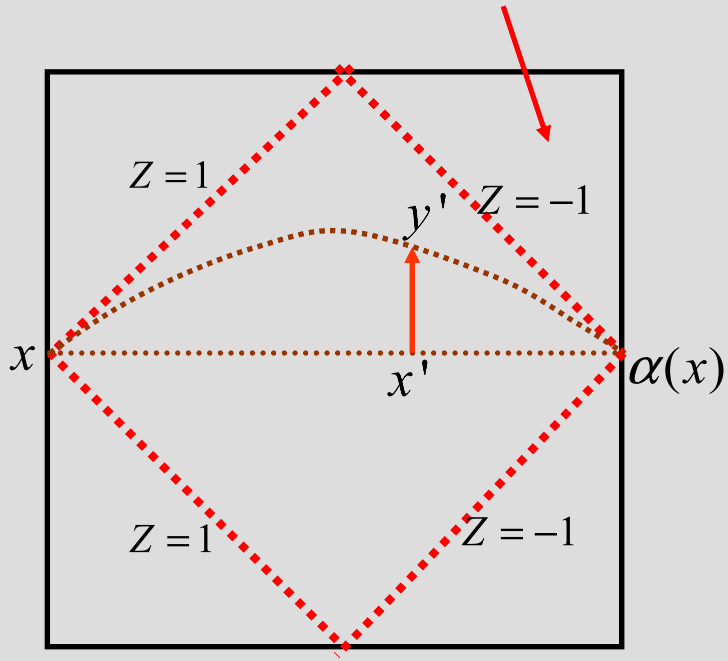
$$\nabla_a n_b = \cot\mu(g_{ab} - n_a n_b),$$
$$\nabla_a n_{b'} = -\csc\mu(g_{ab'} + n_a n_{b'}),$$
$$\nabla_a g_{bc'} = (\csc\mu - \cot\mu)(g_{ab} n_{c'} + g_{ac'} n_b).$$

- geodesic dist for spacelike geod: $\mu = \cos^{-1} Z, \quad -1 \le Z < 1$

  can be def. for **points not connected by geod** $Z < -1$



$(\mu = \chi)$



NOISE KERNEL IN DE SITTER

- The two-point function of the stress tensor operator (noise kernel):

$$N_{abc'd'}(x,x') = \frac{1}{2}\langle 0|\{T_{ab}(x), T_{c'd'}(x')\}|0\rangle - \langle 0|T_{ab}(x)|0\rangle\langle 0|T_{c'd'}(x')|0\rangle$$

in a de Sitter invariant vacuum is an observable maximally symmetric bitensor,which may be written as

$$\begin{aligned} N_{abc'd'} &= P(\mu) n_a n_b n_{c'} n_{d'} + Q(\mu)(n_a n_b g_{c'd'} + n_{c'} n_{d'} g_{ab}) \\ &\quad + R(\mu)(n_a n_{c'} g_{bd'} + n_b n_{d'} g_{ac'} + n_a n_{d'} g_{bc'} + n_b n_{c'} g_{ad'}) \\ &\quad + S(\mu)(g_{ac'} g_{bd'} + g_{bc'} g_{ad'}) + T(\mu) g_{ab} g_{c'd'}. \end{aligned}$$

- The stress tensor operator:

$$T_{ab}(x) = \lim_{x \to x'} \left(g_b^{c'} \nabla_a \nabla_{c'} - \frac{1}{2} g_{ab} g^{cd'} \nabla_c \nabla_{d'} - \frac{1}{2} m^2 g_{ab} \right) \phi(x) \phi(x')$$

NOISE KERNEL IN DE SITTER

- Since the stress tensor can be written as a coincidence limit of the two-point function of the field and its derivatives, the noise kernel is a coincidence limit of a four-point function of the field, which may be written in terms of the Wightman function

$$G^{+}(x,x') = \langle 0 | \phi(x)\phi(x') | 0 \rangle$$

NOISE KERNEL IN DE SITTER

- Impose that vacuum is dS invariant and points spacelike separated. Then Wightman funct is e.v. of observable, and thus dS invariant biscalar ➡ $G^{+} = G(\mu)$
- Wightman funct satisfies KG eq:

$$G''(\mu) + (D-1)\cot\mu\, G'(\mu) - m^2 G(\mu) = 0$$

- Imposing that at coincidence limit diverges as in Minkowski

➡ $$G(\mu) = c_{m,D} F\left(h_+, h_-; \frac{D}{2}; \frac{1+Z}{2} \right)$$

where

$$h_{\pm} = \frac{1}{2}\left((D-1) \pm \sqrt{(D-1)^2 - 4m^2} \right)$$

$$c_{m,D} = \frac{\Gamma(h_+)\Gamma(h_-)}{(4\pi)^{D/2}\Gamma(D/2)}$$

It is the Wightman function of the **Bunch-Davies** vacuum

NOISE KERNEL IN DE SITTER

- Substituting into the noise kernel and using the dS invariant expressions for the covariant derivatives, we obtain the geodesic distance dependent functions of the noise kernel bitensor:

$$P = 2\left(G'' - G'\csc\mu\right)^2,$$

$$Q = -\left(G''\right)^2 + \left(G'\right)^2\left(\csc^2\mu - m^2\right),$$

$$R = -G'\csc\mu\left(G'' - G'\csc\mu\right),$$

$$S = \left(G'\right)^2\csc^2\mu,$$

$$T = \frac{1}{2}\left(G''\right)^2 + \left(G'\right)^2\left(\frac{D-5}{2}\csc^2\mu - m^2\right) + \frac{1}{2}m^4 G^2.$$

NOISE KERNEL FOR SMALL MASSES

- Minimally coupled field, small mass, long distances

$$m \ll 1(=H), \quad Z \ll -1$$

& exp in powers of *m* $\quad G(\mu) = \frac{1}{m^2}\frac{\Gamma(D)}{(4\pi)^{D/2}\Gamma(D/2)} + O(m^0)$

infrared divergence of Bunch-Davies vacuum when *m→0*

$$P, Q, R, S \sim \frac{H^4}{d^4} + O(m^2)$$
$$T \sim const + O(m^2)$$

noise kernel does not vanish at long distances *d*

- In Minkowski ***(**Martin-EV 00)*
here ok short distances (*Z→1*) $\quad P, Q, R, S, T \sim \frac{1}{d^8}$

NOISE KERNEL FOR MASSLESS FIELD

- Minimally coupled field, *m=0*, long distances *d*.
  There is no dS invariant vacuum, choose
  O(D)-inv vacuum (*Allen-Folacci 87*) $|0\rangle_\alpha\,, \quad \alpha \in (0,\infty)$
  It can be shown that (arb close to dS inv)

$$\lim_{\alpha\to 0} \nabla_a \nabla_{b'} G^+_\alpha = \lim_{m\to 0} \nabla_a \nabla_{b'} G^+_{BD}$$

$$\boxed{P,Q,R,S,T \sim \frac{H^4}{d^4}}$$

  vanishes at long distances, discontinuity in limit *m=0*

- Discontinuity is independent of state, at late times

$${}_\alpha\langle 0|T_{ab}|0\rangle_\alpha \to \lim_{\alpha\to 0} {}_\alpha\langle 0|T_{ab}|0\rangle_\alpha$$

  discontinuity disappears if limit in parameter space
  (*Kirsten-Garriga 93*) $\xi R / m^2 \to -2$

RESULTS FOR NOISE KERNEL

- We have derived the stress tensor two-point function of a free quantum field minimally coupled in **dS** in the **BD** state
- For small mass correlations have long range, decay like inverse power of distance $d^{-4m^2/3H^2}$ do not vanish if *m→0*
- For *m=0*, on the contrary, decay like d^{-4} Discontinuity due to infrared divergence of the Bunch-Davies vacuum
- When *H→0* or short range recover Minkowski results

QUANTUM GRAVITATIONAL FLUCTUATIONS

- To characterize **gravitational fluctuations** may compute the two-point function of the linearized **Riemann** tensor. This can be obtained from the two-point functions of the Ricci, or **Einstein**, and the **Weyl** tensors.

- $R_{ab}{}^{cd\,(1)}$ is gauge invariant: Lie derivative wrt arbitrary vector field in **dS** background vanishes.
- Linearized Einstein $G_a{}^{b\,(1)}$ and Weyl $W_{ab}{}^{cd\,(1)}$ are also gauge invariant

- **Tree level** graviton two-point function using dS inv bitensors is known (*Allen-Turyn 87,Antoniadis-Mottola 91*)

QUANTUM GRAVITATIONAL FLUCTUATIONS

E-L eq:

$$G_{ab}^{(1)} + \Lambda h_{ab} - \alpha A_{ab}^{(1)} - \beta B_{ab}^{(1)} = \bar{\kappa} \langle \hat{T}_{ab}^{(1)} \rangle_{ren} + \bar{\kappa} \xi_{ab}[g]$$

- After **order reduction** for higher derivative terms: in **dS** the counterterms do not contribute $A_{ab}^{(1)}[dS] = B_{ab}^{(1)}[dS] = 0$

- The linearized **Einstein tensor two-point correlation function** directly related to **noise kernel**

$$\langle G_b^{a(1)}(x) G_{d'}^{c'(1)}(x') \rangle_c \equiv \frac{1}{2} \langle \{ G_b^{a(1)}(x), G_{d'}^{c'(1)}(x') \} \rangle - \langle G_b^{a(1)}(x) \rangle \langle G_{d'}^{c'(1)}(x') \rangle$$

$$= \frac{(8\pi)^2 \bar{l}_P^4}{N} N^{a\ c'}_{\ b\ d'}(x, x')$$

QUANTUM GRAVITATIONAL FLUCTUATIONS

- Massless $m = 0, \quad \xi = 0$
  long distances $d \gg H^{-1}$

$$\langle G_b^{a(1)}(x) G_{d'}^{c'(1)}(x') \rangle_c \sim \frac{\bar{l}_P^{\,4} H^4}{N d^4}$$

- Low mass $m \ll H, \quad \xi = 0$
  long distances
  i.e. long range correlations

$$\langle G_b^{a(1)}(x) G_{d'}^{c'(1)}(x') \rangle_c \sim \bar{l}_P^{\,4} H^8 N^{-1} \left(Hd \right)^{-\frac{4m^2}{3H^2}}$$

- The two-point function for linerized Weyl tensor has been computed at tree level (*Kouris 01*)
  need **loop corrections**

$$\langle W_{abcd}^{(1)}(x) W_{a'b'c'd'}^{(1)}(x') \rangle \sim \frac{\bar{l}_P^{\,2} H^2}{N d^4} + O\left(\bar{l}_P^{\,4} H^4 \right)$$

MINKOWSKI AND SHORT DISTANCES DE SITTER

- In **Minkowski** (*Martin-EV 00*),
  also **dS** short distances take $H \to 0$

$$\langle G_b^{a(1)}(x) G_{d'}^{c'(1)}(x') \rangle_c = (8\pi)^2 \frac{\bar{l}_P^4}{N} N^{a\ \ c'}_{\ b\ \ d'}(x,x') \sim \frac{\bar{l}_P^4}{N d^8}$$

$$\langle W_{abcd}^{(1)}(x) W_{a'b'c'd'}^{(1)}(x') \rangle \sim \frac{\bar{l}_P^2}{N d^6}$$

RESULTS

- Our results give information on the quantum fluctuations of the gravitational field in **dS** including matter loops.

- Neglect contribution from graviton loops. Implemented in the context of **large** *N*, rescaling $\bar{l}_P = l_P \sqrt{N}$ and expanding in *1/N*, or equivalently, **stochastic gravity**.

RESULTS

- **All information** is in the Riemann tensor two-point function. Obtained from the Ricci (or Einstein) and Weyl tensors.
- The linearized Einstein tensor two-point f. directly related to stress tensor correlations, includes matter one-loop order.
- Weyl to one-loop needs to be computed, tree level is known.
- So far no evidence of breaking dS
- Long range correlations for low mass fields

MASSLESS LIMIT

- The field operator in spatially flat coordinates,

$$\hat{\phi}(t,\vec{x}) = \int d^3\vec{k}\left(\hat{a}_k u_k(t) e^{i\vec{k}\cdot\vec{x}} + \hat{a}_k^\dagger u_k^*(t) e^{-i\vec{k}\cdot\vec{x}}\right)$$

Klein-Gordon eq leads to

$$\ddot{u}_k + 3H\dot{u}_k + \left(k^2/a^2 + m^2\right)u_k = 0$$

and the Wightman function at equal times:

$$G^+(x,x') = \langle 0|\hat{\phi}(t,\vec{x})\hat{\phi}(t,\vec{x}')|0\rangle = \int_0^\infty dk k^2 \frac{\sin kr}{kr}|u_k(t)|^2$$

$r = |\vec{x} - \vec{x}'|$

- Using conformal time $a\,d\eta = dt$ and dS

$$a = \frac{-1}{H\eta} = e^{Ht}$$

KG eq is

$$u_k'' - \frac{2}{\eta}u_k' + \left(k^2 + \frac{m^2}{H^2\eta^2}\right)u_k = 0$$

SMALL MASS

- Then defining $u_k \equiv \frac{1}{a(\eta)} \chi_k$ $a(\eta) = \frac{-1}{H\eta}$

  KG eq becomes $$\chi_k'' + \left(k^2 - \left(2 - \frac{m^2}{H^2}\right)\frac{1}{\eta^2}\right)\chi_k = 0$$

- At early times (inside): $k|\eta| >> 1$ $\chi_k(\eta) \approx \frac{1}{\sqrt{2k}} e^{-ik\eta}$ **BD vacuum**

- At late times (out): $k|\eta| << 1$ $\chi_k(\eta) = A_k|\eta|^{n_1} + B_k|\eta|^{n_2}, \quad n_{1,2} \equiv 1/2 \pm \sqrt{9/4 - m^2/H^2}$

  -ve exp dominates $\chi_k(\eta) \approx A_k|\eta|^{-1+m^2/3H^2}$

- Match at **horizon exit** $k|\eta_k| = 1 \Rightarrow k = a(\eta_k)H$ $A_k \sim k^{-3/2+m^2/3H^2}$

$$u_k(\eta) \approx HA_k|\eta|^{m^2/3H^2} = \frac{H}{k^{3/2}}\left(\frac{k}{aH}\right)^{m^2/3H^2}$$

SMALL MASS

- For physical distances larger than horizon

$$d = ar \gg H^{-1}$$

- Finally the Wightman function
  (use dimensionless variable: kr):

$$G^{+}(t,\vec{x};t,\vec{x}') \simeq \frac{H^4}{m^2}\left(Hd\right)^{-2m^2/3H^2}$$

and the **stress tensor two-point funct** which depends on

$$T = \ldots + \frac{1}{2} m^4 G^2$$

decays like $d^{-4m^2/3H^2}$

i.e. in **massless limit** is a constant at long distances

DE SITTER SPACETIME

- Maximally symmetric spacetime, isometry group *O(D,1)*
- Hyperboloid in (*D+1*)-Minkowski (points at const dist form origin)

$$-\left(X^0\right)^2+\left(X^1\right)^2+\ldots+\left(X^D\right)^2=H^{-2}$$

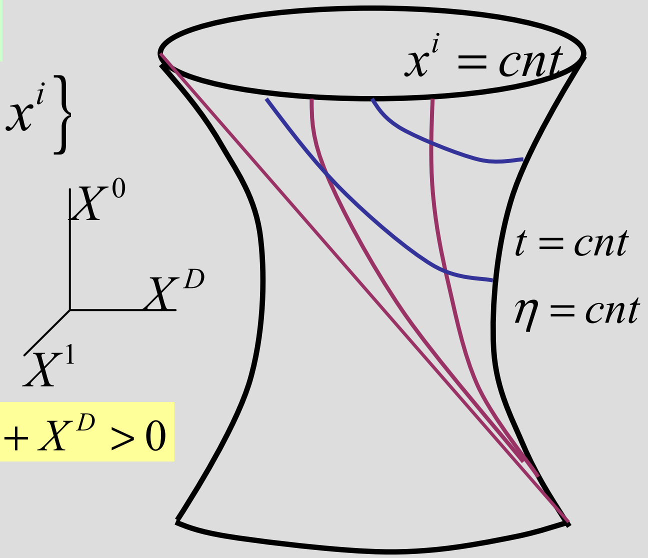
$$ds^2=-\left(dX^0\right)^2+\left(dX^1\right)^2+\ldots+\left(dX^D\right)^2$$

- Introduce spatially flat coordinates $\{t, x^i\}$

$$X^0=\frac{1}{H}\sinh(Ht)+\frac{1}{2}He^{Ht}\delta_{ij}x^i x^j$$

$$X^D=\frac{1}{H}\cosh(Ht)-\frac{1}{2}He^{Ht}\delta_{ij}x^i x^j$$

$$X^i=x^i e^{Ht};\quad i=1,\ldots,D-1;-\infty<t,x_i<\infty$$



$$X^0+X^D>0$$

- Metric

$$ds^2=-dt^2+e^{2Ht}\delta_{ij}dx^i dx^j=a^2(\eta)\left(-d\eta^2+\delta_{ij}dx^i dx^j\right)$$

$$a=\frac{-1}{H\eta}=e^{Ht}$$

conformal time $a\,d\eta=dt$

DS AND COSMOLOGICAL CONSTANT

- Metric in dS is a sol of Einstein eq:

$$G_{ab} = -\Lambda g_{ab} \qquad \Lambda = [(D-1)(D-2)/2]H^2$$

- Is a FRW model with a **cosmological constant** Λ

  It can be seen as a perfect fluid with $p = -\varepsilon$

$$T_{ab} = (\varepsilon + p)u_a u_b - p g_{ab} \equiv \varepsilon_\Lambda g_{ab}$$

- Spatially flat coordinates do not cover the dS spacetime.

  With conformal time $\eta = -\int_t^\infty a^{-1}(t)dt = -H^{-1}\exp(-Ht)$; sph coor (*D=4*)

$$ds^2 = \frac{1}{H^2\eta^2}\left(-d\eta^2 + dr^2 + r^2(d\theta^2 + \sin^2\theta d\varphi^2)\right), \quad -\infty < \eta < 0, \quad 0 \le r < \infty$$

- Define new coord $\eta = \dfrac{\sin\tilde{\eta}}{\cos\tilde{\eta} + \cos\chi}, \quad r = \dfrac{\sin\chi}{\cos\tilde{\eta} + \cos\chi}$

NOISE KERNEL IN DE SITTER

- Since the stress tensor can be written as a coincidence limit of the two-point function of the field and its derivatives, the noise kernel is a coincidence limit of a four-point function of the field, which may be written in terms of the Wightman function

$$G^{+}(x,x') = \langle 0|\phi(x)\phi(x')|0\rangle$$

- Finally the noise kernel becomes

$$\begin{aligned} N_{abc'd'} &= \nabla_a \nabla_{c'} G^+ \nabla_b \nabla_{d'} G^+ + \nabla_a \nabla_{d'} G^+ \nabla_b \nabla_{c'} G^+ \\ &- g_{ab}\left(\nabla_e \nabla_{c'} G^+ \nabla^e \nabla_{d'} G^+ + m^2 \nabla_{c'} G^+ \nabla_{d'} G^+\right) \\ &- g_{c'd'}\left(\nabla_a \nabla_{e'} G^+ \nabla_b \nabla^{e'} G^+ + m^2 \nabla_a G^+ \nabla_b G^+\right) \\ &+ \frac{1}{2} g_{ab} g_{c'd'}\left(\nabla_e \nabla_{e'} G^+ \nabla^e \nabla^{e'} G^+ + m^2\left(\nabla_e G^+ \nabla^e G^+ + \nabla_{e'} G^+ \nabla^{e'} G^+\right) + m^4 G^{+2}\right) \end{aligned}$$