Onset of geodesic chaos in the black hole - disc system

Petra Suková, Oldřich Semerák

Institute of Theoretical Physics
Faculty of Mathematics and Physics
Charles University in Prague

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Motivation

- indirect evidence of black holes
  - galactic nuclei
  - X-ray binaries

  $\rightarrow$ not isolated black hole
- the presence of additional matter – influence mainly on higher derivatives of the metric
  - configuration and stability of the external source
  - geodesic motion of test particles – possibility of chaotic motion
- interaction of the BH and surroundings $\Rightarrow$ disc accretion
- accretion models – mostly only test discs – is this a good approximation?
- Chaotic systems studied in the literature
  - infinite thin disc with inner rim and constant mass density – pseudo-Newtonian potential and full general relativistic solution
  - halo with multipole contributions
  - rotating black holes – approximate solutions

Our systems of interest

- exact solutions (nonlinear GR effects play role)
- initially Schwarzschild black hole – central source of field
  $+$ annular thin disc or thin ring – perturbation
  - family of inverted Morgan-Morgan discs (iMM discs)
    (Morgan & Morgan, 1969; Lemos & Letelier, 1994; Semerák & Žáček, 2000)
  - family of discs with power-law density profile (PL discs)
    (Semerák, 2004)
    potential smoother at the inner rim than iMM discs, but much more computer-time demanding
  - Bach-Weyl rings
    (Bach, Weyl, 1922)
    more singular source – one-dimensional

finite total mass $\mathcal{M}$
mass concentrated near the inner rim

Inverted Morgan-Morgan discs

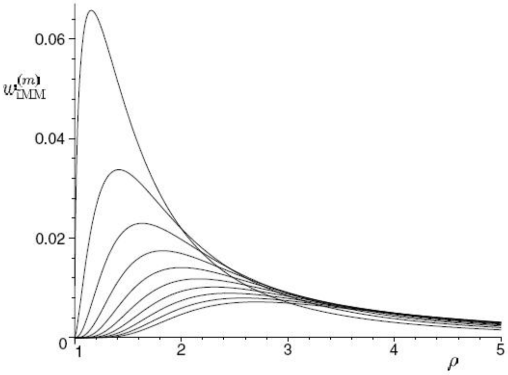


Figure 1. Radial course of the Newtonian density (12) of the inverted MM counter-rotating discs. The discs with $m = 1, 2, 3, \ldots, 10$ are included; in this order, the density becomes smoother at the inner rim (which is fixed to unit Weyl radius) and has a less distinct maximum, of $\frac{3^{3/2} \cdot 2^{m-1} (m!)^2 (2m-1)^{m-1/2} \mathcal{M}}{\pi^2 (2m)! (m+1)^{m+1} b^2}$, lying at greater radius, $\rho_{\max} = \sqrt{(2/3)(m+1)}\, b$. The horizontal axis (Weyl radius ρ) is in the units of disc mass $\mathcal{M}$, while the vertical axis ($w_{\mathrm{iMM}}^{(m)}$) is in the units of $\mathcal{M}^{-2}$.

from (Semerák, 2003)

Constants of geodesic motion

- generally 3 degrees of freedom
  (four-velocity fulfil normalization $g_{\mu\nu}u^{\mu}u^{\nu} = -1$)
- static axially symmetric system
  - two Killing vectors
  - two isolating integrals
    - energy per unit mass at infinity
      $E = -g_{tt}u^t$
    - angular momentum per unit mass at infinity
      $L = g_{\phi\phi}u^{\phi}$
- no other isolating integral due to any symmetry (no irrecudible Killing tensor: Walker, Penrose, 1970)
  $\longrightarrow$ possibility of chaotic motion
- our goal: describe the onset of chaos in the system in
  dependence on parameters
  compare the influence of different external sources on the phase space
  compare different methods of recognizing chaos

Methods

- Poincare's surface-of-section method
  phase space topology depends on parameters of the disc ($\mathcal{M}$, b) and of the particle (E, L)
- latitudinal action

$$J_\theta = \frac{1}{I} \int \sqrt{g_{\theta\theta} u^\theta u^\theta} \mathrm{d}\tau$$

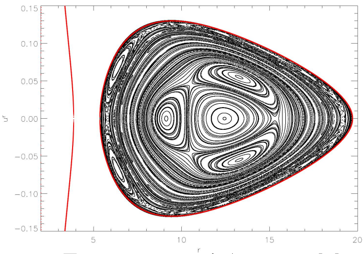
- characteristics of time series of a single quantity
  - power spectrum of the particle's vertical position, $z = z(t)$

$$z(\omega) = \lim_{T \to \infty} \int_0^T z(t) \mathrm{e}^{\mathrm{i}\omega t} \mathrm{d}t$$

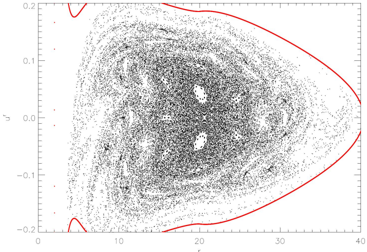
$$P(\omega) = |z(\omega)|^2$$

  - weighted average of directional vectors with varying τ (Kaplan, Glass, 1992)

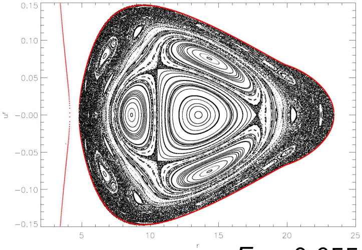
Poincare's surface of section – $L = 3.75M$, $b = 20M$

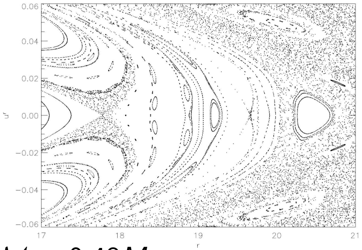


$E = 0.952$, $\mathcal{M} = 0.5M$



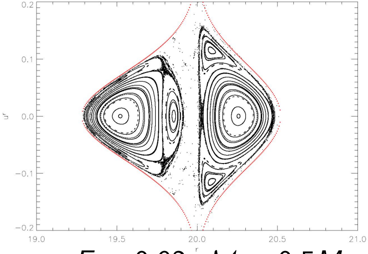
$E = 0.955$, $\mathcal{M} = 1.1M$



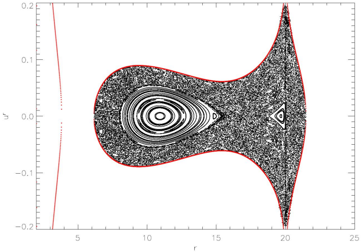


$E = 0.955$, $\mathcal{M} = 0.48M$

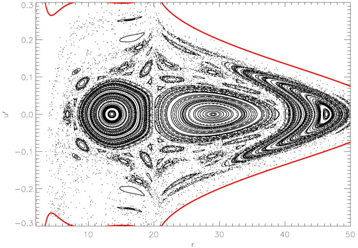
Bach-Weyl ring with $L = 3.75M$, $b = 20M$



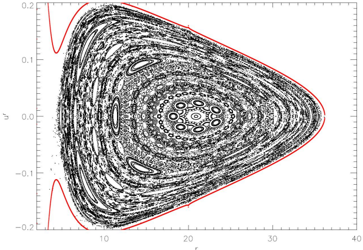
$E = 0.92$, $\mathcal{M} = 0.5M$



$E = 0.93$, $\mathcal{M} = 0.5M$



$E = 0.975$, $\mathcal{M} = 0.5M$

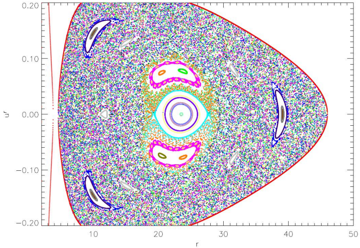


$E = 0.977$, $\mathcal{M} = 0.006M$

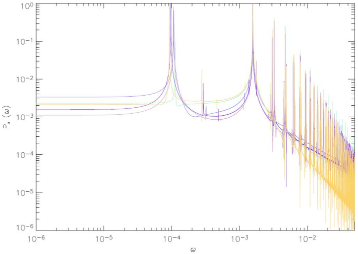
more results (Semerák, Suková, 2010)

Power spectra - $E = 0.956$, $L = 4M$, $b = 20M$, $\mathcal{M} = 1.3M$

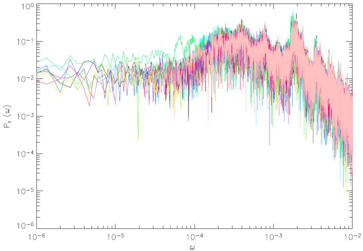
Poincare's surface of section



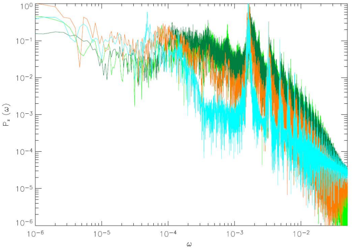
regular orbits



orbits from "chaotic sea"



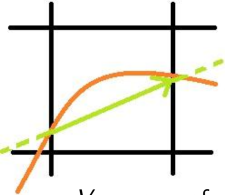
sticky motion



sticky motion = $1/f$-dependence (Koyama, Kiuchi, Konishi, 2007)

Average directional vectors

- input data – time series of some dynamical quantity (experimentally measurable)
  in our case: $z(t)$ – coordinate-time dependence of the z-component of particles' position
- reconstruction of 3D phase space by time delay (making another two copies of the series by shifting it twice by a chosen time tau)
  trajectory coordinates: $z(t)$, $z(t-\tau)$, $z(t-2\tau)$



- dividing the phase space into k x k x k grid
- determining the normalized directional vector of the trajectory passing through the box
- weighted average $\Lambda = \left\langle \frac{(V_j/n_j)^2 - (R^d_{n_j})^2}{1-(R^d_{n_j})^2} \right\rangle$

V_j – norm of vector addition of all directional vectors in box j, $d = 3$
R^d_m – average displacement per step for random walk in d dim. for m steps,
n_j – number of passages through box j

Time series analysis – $E = 0.995$, $L = 3.75M$, $b = 20M$, $\mathcal{M} = 0.5M$