

# Non-axisymmetric Torsional Oscillations of Relativistic Stars

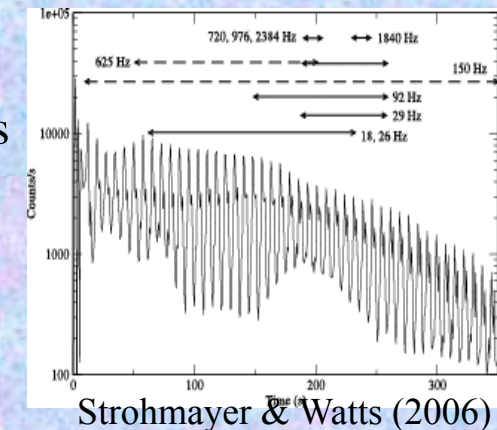
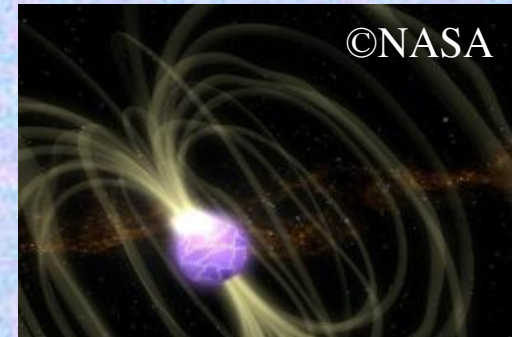
**Hajime SOTANI**

**National Astronomical Observatory of Japan**

Collaborated with  
K.D.Kokkotas in Tuebingen

# Observations

- Soft gamma Repeaters and Anomalous X-ray Pulsars are candidates of **Magnetars**, which are neutron stars with strong magnetic fields.
- **Soft gamma Repeaters** (SGRs)
  - radiating sporadic X- and gamma-ray bursts ( $\sim 10^{41}$  erg/s)
- **Giant Flare from SGRs ( $10^{44}$ - $10^{46}$  ergs/s)**
  - SGR 0526–66 in March.5.1979
  - SGR 1900+14 in August.27.1998
  - **SGR 1806–20** in December.27.2004
- In the decaying tail after the flare, **QPOs** are found !!
  - Barat et.al. (1983); Israel et.al. (2005);  
Watts & Strohmayer (2005, 2006)
  - SGR 0526-66 : **23ms (43Hz)**,  $B \sim 4 \times 10^{14}$ G,  $L \sim 10^{44}$  ergs/s
  - SGR 1900+14 :  $B > 4 \times 10^{14}$ G, **28, 54, 84, and 155 Hz**
  - SGR 1806–20 :  $B \sim 8 \times 10^{14}$ G,  $L \sim 10^{46}$  ergs/s  
**18, 26, 30, 92.5, 150, 626.5, and 1837 Hz**  
**(also 720Hz ?? and 2384 Hz ??)**



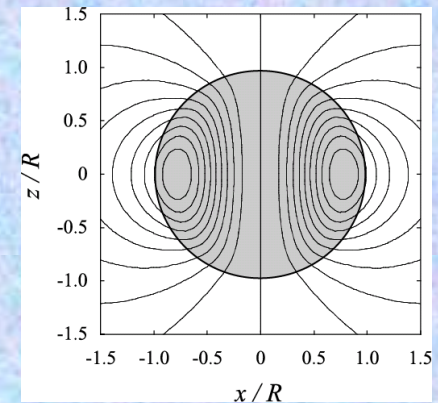
**Final goal of our study is to explain these observed evidences theoretically.**

# *Models of Magnetar*

- Ideal MHD approximation
  - Electric fields are zero for comoving observer.
- The stellar deformation due to the magnetic fields are neglect.
  - Magnetic energy / gravitational energy  $\sim 10^{-4} (B/10^{16}[G])^2$
  - Equilibrium configuration : static spherically symmetric
- Axisymmetric poloidal magnetic fields

## *Perturbations*

- Linearizing the equation of motion and Maxwell equations
  - Cowling approximation ( $\delta g_{\mu\nu} = 0$ )



# How to explain QPOs - I

- **QPOs of SGRs are due to the crust torsional oscillations ??**

- In Newtonian; Hansen & Cioffi (1980), McDermott et al. (1998), Carroll et al. (1986), Storchmayer (1991), ...

→ the case without magnetic fields

$$\ell t_0 \sim \frac{\sqrt{\ell(\ell+1)\mu/\rho}}{2\pi R} \sim 16\sqrt{\ell(\ell+1)} \text{ Hz} \quad \ell t_n \sim \frac{\sqrt{\mu/\rho}}{2\Delta r} \sim 500 \times n \text{ Hz}$$

→  $2t_0 = 39$ ,  $3t_0 = 55$ ,  $4t_0 = 72$ ,  $5t_0 = 88$ ,  $6t_0 = 104$ ,  $\dots$ ,  $\ell t_1 = 500$ ,  $\dots$

- In GR; Schumaker & Thone (1983), Leins (1994), Samuelsson & Andersson (2006), Sotani, Kokkotas & Stergioulas(2007)

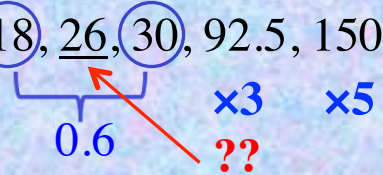
- **This attempt might be *partially* successful.**

- The stellar models with stiff EOS and massive star are favored.
- **However, it is found the difficulty to explain all observed frequencies of QPOs.**
- Explanation for lower frequencies could be impossible with only using the crust torsional oscillations.

→ Observed frequencies in SGR 1806-20; 18, 26, and 30Hz

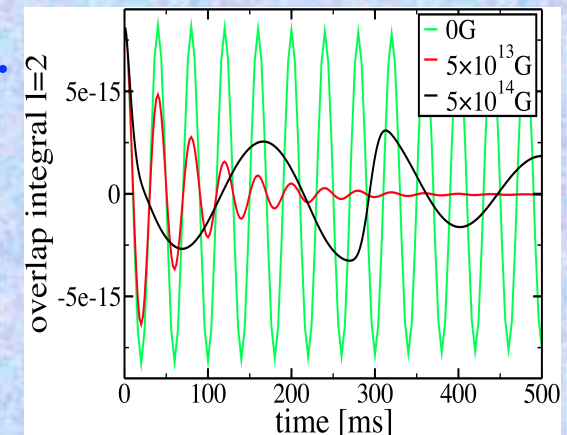
→ *The interval of observed frequencies is much smaller than that expected by the torsional oscillation with different values of  $l$ .*

# How to explain QPOs - II

- **Alfven oscillations in the core region ??**
  - Levin(2006), Glampedakis et al. (2006)
  - Levin(2007) : the QPO frequencies are enhanced at their edges or turning point.
  - Sotani et al. (2008) : find the two families in Alfven QPOs
  - Colaiuda et al. (2009), Cerda-Duran et al. (2009) : more detailed studies
- Time evolution of two dimensional wave equations
  - Spectrum of Alfven oscillation becomes *continuum*.
  - There exist two families; *upper and lower QPOs*.
  - $f_{Ln} \approx 0.6 \times f_{Un}$ ,  $f_{Un} \approx (n+1) \times f_{U0}$ ,  $f_{Ln} \approx (n+1) \times f_{L0}$ ,  $f \propto B$ .
- Observed frequencies of QPOs in SGRs
  - SGR 1806-20 : 18, 26, 30, 92.5, 150 Hz
    - 
  - *Still, it seems to be difficult to explain all observed frequencies.*

# Magneto-Elastic Oscillations

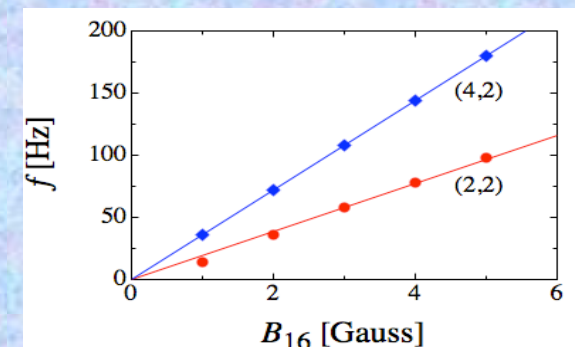
- Gabler et al. (2010) make a numerical simulations of axisymmetric torsional Alfvén oscillations in magnetars including the crust region.
  - With weaker magnetic strength, one can observe only the crust oscillation.
  - With stronger magnetic strength, only Alfvén oscillations can be observed.
  - The critical magnetic strength for exchanging the oscillation type is  $B \approx 4 \times 10^{14}$  G ??
- Thus, the axisymmetric axial type oscillations in the magnetar could become (i) crust oscillations or (ii) Alfvén oscillations
  - **One might not observe the both types of oscillation.**
- To explain the observed evidences, alternative types of oscillation should be considered.
  - Polar type oscillations ?
    - Sotani et al. (2009) suggested that the typical frequencies in magnetars are around 100 Hz.
  - **Non-axisymmetric oscillations ?**



Gabler et al. (2010)

# Non-axisymmetric Oscillations

- The axial oscillations are coupled with the polar ones even for non-rotating magnetars.
  - As a first step, we consider the only axial type oscillations.
- We find that...
  - **non-axisymmetric axial Alfvén oscillations are discrete oscillations.**
    - It could be excited that the both crust and Alfvén oscillations ??
  - **Those frequencies are smaller than that of axisymmetric axial type Alfvén oscillations.**
    - Axisymmetric case; minimum frequency is around 15 Hz for  $B=4\times 10^{15}\text{G}$
    - Non-axisymmetric case;  $f_{22}=7.7$  and  $f_{42}=14.4\text{Hz}$  for  $B=4\times 10^{15}\text{G}$
- This type of oscillations could be important to explain theoretically the observed evidence of QPOs in the SGR
  - To fit the possible stellar model with the observations in SGRs, it is necessary to produce more oscillation frequencies with different value of  $(l,m)$  for the stellar models constructed with different EOSs.



# *Conclusion*

- QPOs are found in the SGRs.
- As well as crust torsional oscillations, the axial Alfvén oscillations might be partially successful to explain the observed frequencies, still those are impossible to explain the all.
- In more realistic stellar model, in which the coupling between the core and crust region will be considered, the axisymmetric torsional oscillations become crust type or Alfvén type of oscillations.
  - One needs to consider another type of oscillations.
- Non-axisymmetric, axial type Alfvén oscillations becomes discrete spectrum.
- The typical frequencies of this type of oscillations become smaller than those for the axisymmetric torsional oscillations.
- Non-axisymmetric oscillations could be important to explain the observed evidences.



# *Future works*

- Dependence of the toroidal magnetic field on the oscillation frequency ??
- Introducing the effect of crust region.
- Coupling between the axial and polar oscillations.

*Thanks for your attentions*