Closed trapped surfaces in higher dimensional self-similar Vaidya spacetime

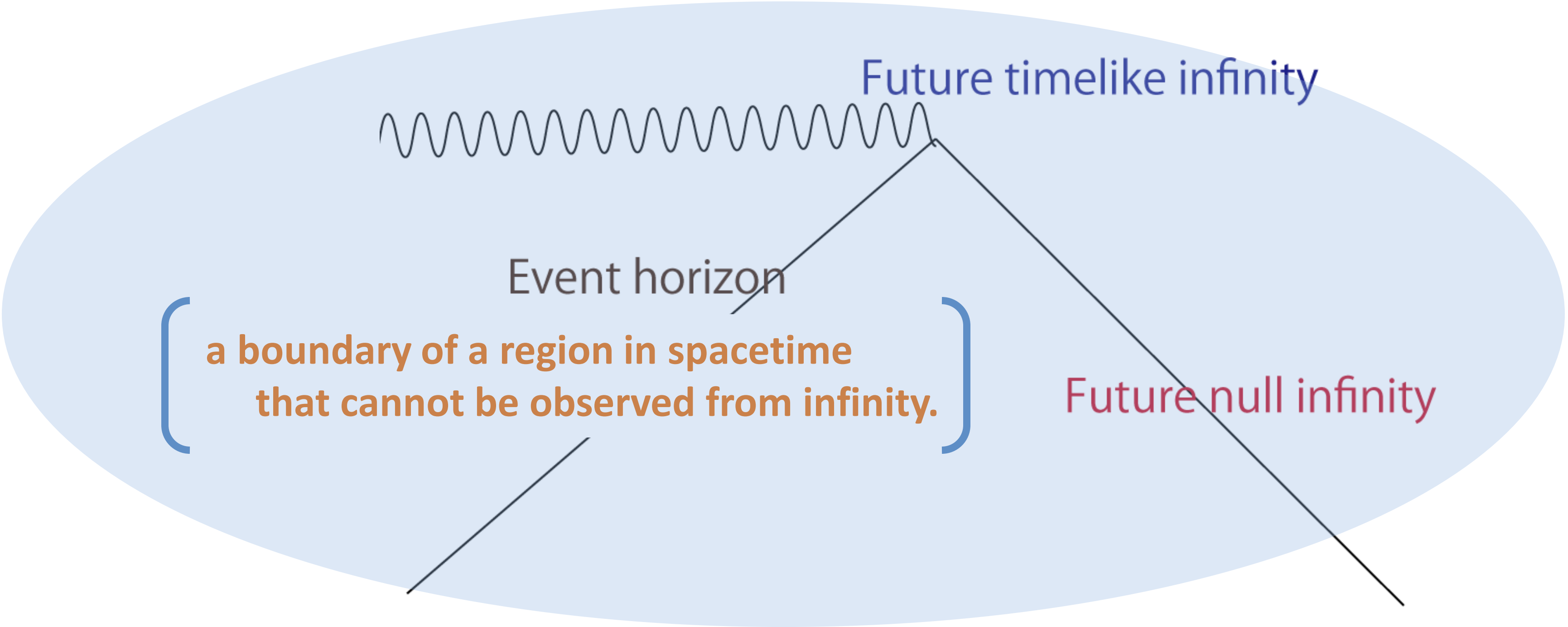
Masahiro SHIMANO

Rikkyo Univ. Japan

collaboration with Tomohiro HARADA

What is black hole

A black hole is defined by **an event horizon**



- ✓ Unless we know an entire future evolution of spacetime, we cannot define black holes.
- ✓ It might be difficult to study dynamical black holes by this definition .

⇒ **We would like to define black holes constructively**

Toward constructive definition of Black hole

By using the following surfaces, it might be possible to define black hole constructively

- **Trapped surfaces**

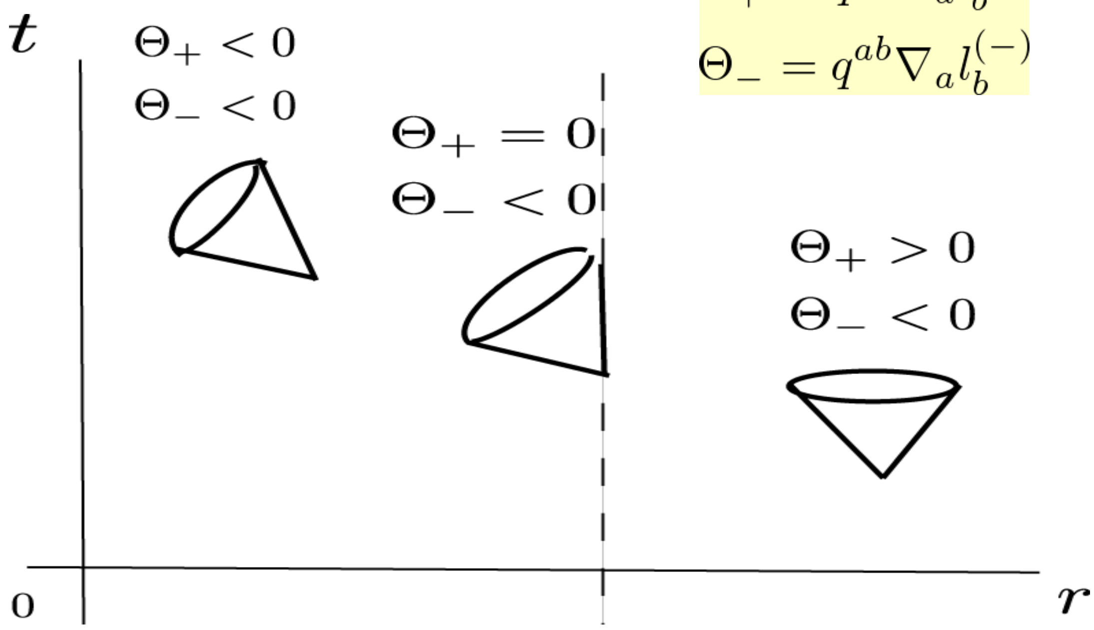
  closed D-2 surfaces whose boss null expansions are negative.

- **Trapped regions**
- **Apparent horizons**

Etc.

$$\Theta_+ = q^{ab}\nabla_a l_b^{(+)}$$

$$\Theta_- = q^{ab}\nabla_a l_b^{(-)}$$



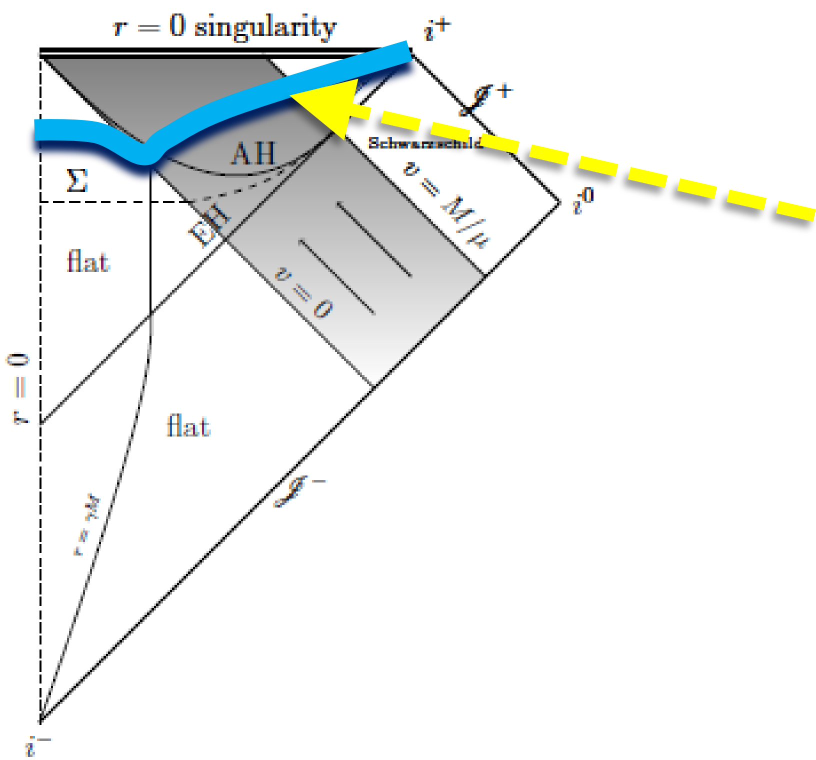
Trapped surfaces in Vaidya spacetime

Recently, trapped surfaces in Vaidya spacetime have interesting facts.

✓ **Trapped surfaces are extended into the flat region in Vaidya spacetime**

E. Schnetter and B. Krishnan, Phys. Rev, D 73, 021502(R) (2006).

I. Bengtsson, and J. M. M. Senovilla, Phys. Rev. D79 (2009) 024027.



Trapped surfaces

Trapped surfaces are extended into the flat region (Trapped surfaces are extended outside the apparent horizon)

Why higher dimensional spacetime?

Recently, it is discussed that **Higher dimensional black hole and naked singularity may be detected at the LHC.**

- In the framework of the large extra-dimension scenario, substantial **black holes may be generated at the CERN Large Hadron Collider (LHC)**.

  S. B. Giddings and S. Thomas, Phys. Rev. D 65, 056010 (2002).

- Not only black holes but also **the naked singularity may be generated at the LHC** in the framework of the large extra-dimension scenario**(←See Miyamoto's poster)**.

  K. Nakao, T. Harada and U. Miyamoto, arXiv:1007.4610v1.

Objective of this study

We apply the construction method of trapped surfaces discussed by Bengtsson and Senovilla into higher dimensional self-similar Vaidya spacetime, and will show the following result :

1. **Trapped surfaces are extended into the flat region,**
2. **If spacetime has these trapped surfaces, there never occur the naked singularity.**

$$ds^2 = -\left(1 - \frac{2m}{nr^n}\right) dv^2 + 2dvdr + r^2 \left(d\theta_1^2 + \sin^2\theta_1 d\theta_2^2 + \cdots + \sin^2\theta_1 \cdots \sin^2\theta_n d\theta_{n+1}^2\right)$$

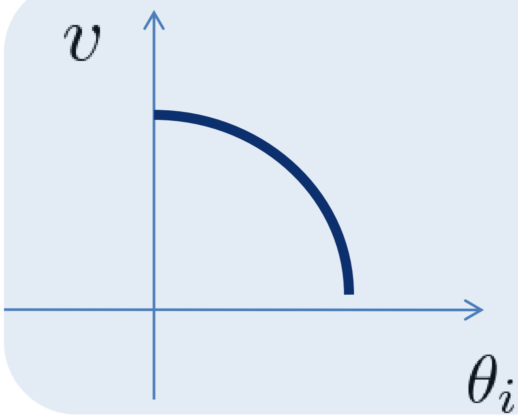
where $n = D - 3$

$$m = \begin{cases} 0 & , v \leq 0 \\ \mu v^n & , 0 \leq v \leq M^{1/n}/\mu \\ M & , v \geq M^{1/n}/\mu \end{cases}$$

Two kinds of n+1-surfaces

We match two kinds of n+1-surfaces :

Constant radius surface(CRS) and **Constant zenith angle surface(CZS)**

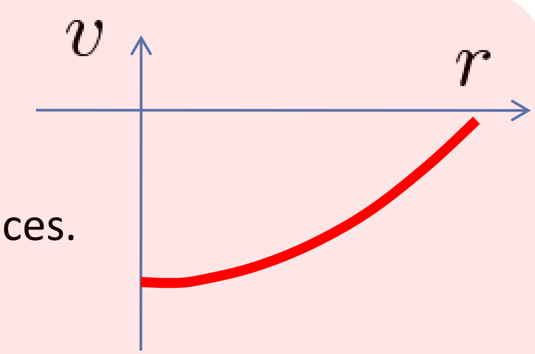


CRS (t and θ_i are functions of ρ, r =const.)

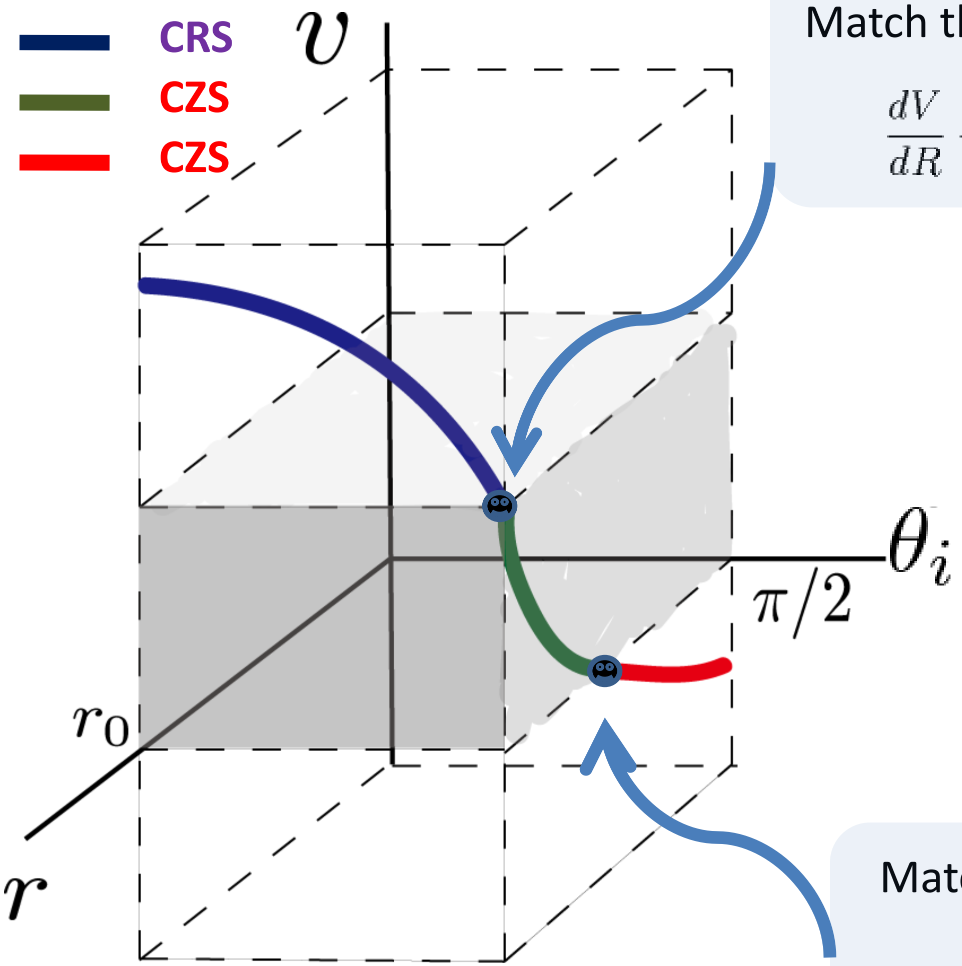
✓Bend surfaces into **quadrant of the circle.**
✓Impose the condition on parameters, to obtain **negative both null expansions**.

CZS (t and r are functions of ρ, $\theta_i = \pi/2$)

✓By using **the hyperboloids function** we bend surfaces.
✓In this case, **both null expansions are negative.**



Match the surfaces



Match the derivative

$$\frac{dV}{dR} \to \infty.$$

✓To match surfaces smoothly we use [green line] which has the derivative

$$\frac{dV}{dR} = \frac{a}{b - X}, \quad \text{where} \quad X = \frac{V}{R}$$

✓Impose constraints on parameters to obtain negative both null expansions.

Match the derivative

$$0 < \frac{dV}{dR} < 1.$$

Result

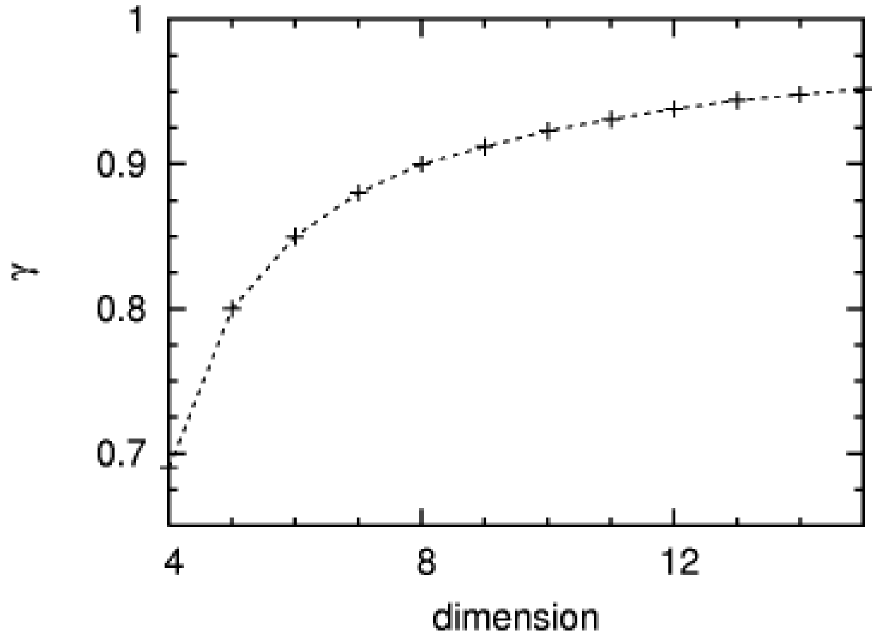
- The case of $\theta_1 = \pi/2$

If mass parameter satisfies the following condition, Trapped surfaces are extended into the flat region.

$$\mu > \frac{1}{4}\left(\frac{n}{\gamma}\right)^{1/n} \qquad \gamma = \frac{n r_0^n}{M} < 1$$

γ must satisfy the condition :

$$n\sqrt{\frac{2}{\gamma}-1}\left(\frac{1}{\gamma}-1\right) > \frac{2}{\pi}$$



Trapped surfaces are extended into the flat region (and extended outside the apparent horizon).

Naked singularity

Spacetime has the naked singularity, if mass parameter satisfies the condition

$$\mu < \left[\frac{(D-3)}{2(D-2)}\right]^{D-2}$$

Spacetime has trapped surfaces, if mass parameter satisfies the condition

$$\mu > \frac{1}{4}\left(\frac{n}{\gamma}\right)^{1/n}$$

These conditions are inconsistent

If spacetime has trapped surfaces such as have been constructed in this study, **there never exist naked singularity.**

Summary

- In order to understand dynamical black holes, it might be important to define black holes with **constructive notion,** for example, trapped surfaces.
- In higher dimensional self-similar Vaidya spacetime, we have applied Bengtsson and Senovilla's method, and have shown the following results :
  - ✓ **trapped surfaces are extended into the flat region**,
  - ✓ **the naked singularity never occur.**
- In other spacetime, we will consider trapped surfaces such as have been discussed in this study (In preliminary, we have constructed these in Oppenheimer-Snyder spacetime).