Charged Rotating Black Holes in Higher Dimensions

Masoud Allahverdizadeh[1], Jutta Kunz[1]
Francisco Navarro-Lérida[2]

[1] Institut für Physik CvO Universität Oldenburg
[2] Depto. Física Atómica, Molecular y Nuclear, UCM

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Outline

- Introduction
- Einstein-Maxwell-Dilaton Black Holes
- Numerical method
- Perturbative method
- Conclusions

Introduction

- 4D Einstein-Maxwell (EM) black holes

	Static	Rotating
Uncharged	Schwarzschild (M)	Kerr (M, J)
Charged	Reissner-Nordström (M, Q, P)	Kerr-Newman (M, J, Q, P)

- D>4 Einstein-Maxwell black holes

	Static	Rotating
Uncharged	Tangherlini (M)	Myers-Perry (M, J_i)
Charged	Tangherlini (M, Q)	?

Introduction

- Problems to find black holes in higher dimensions: more complicated equations to solve with larger number of coordinates.
- Additional problem: the topology of the horizon is not unique (Black rings are allowed for D>4 Emparan&Reall 2002)
- One has to use symmetries to simplify the problem: spacetime symmetries.
- One has to concentrate on a concrete topology for the horizon.
- Aim: Higher dimensional Abelian (=with a Maxwell field coupled) black holes asymptotically flat and with regular horizon (with spherical topology). Stationary axisymmetric solutions.

Introduction

- D-dimensional spacetime
  1 timelike coordinate + (D-1) spatial coordinates
- Odd D (=2N+1)
  2N spatial coordinates $\longrightarrow$ N orthogonal spatial planes $\longrightarrow$ N independent angular momenta J_i , i=1,…,N
- Even D (=2N+2)
  2N+1 spatial coordinates $\longrightarrow$ N orthogonal spatial planes $\longrightarrow$ N independent angular momenta J_i , i=1,…,N
- General stationary axisymmetric solutions will depend on D-(N+1) spatial coordinates: very complicated for high D
- Simplification for odd D: If $|J_i| = J$ (same magnitude for all angular momenta), isometry group enlarges (from $R\times U(1)^N$ to $R\times U(N)$)
  Consequence: all the angular dependence may be analytically extracted!!!
- One has to deal with ODE's only!!!

Introduction

- Ansätze (D=2N+1) (all functions depend on r only)

$$ds^2 = -f dt^2 + \frac{m}{f}\left[dr^2 + r^2 \sum_{i=1}^{N-1}\left(\prod_{j=0}^{i-1}\cos^2\theta_j\right) d\theta_i^2\right]$$

$$+\frac{n}{f}r^2 \sum_{k=1}^{N}\left(\prod_{l=0}^{k-1}\cos^2\theta_l\right)\sin^2\theta_k\left(\varepsilon_k d\varphi_k - \frac{\omega}{r}dt\right)^2$$

$$+\frac{m-n}{f}r^2\left\{\sum_{k=1}^{N}\left(\prod_{l=0}^{k-1}\cos^2\theta_l\right)\sin^2\theta_k d\varphi_k^2\right.$$

$$\left.-\left[\sum_{k=1}^{N}\left(\prod_{l=0}^{k-1}\cos^2\theta_l\right)\sin^2\theta_k\varepsilon_k d\varphi_k\right]^2\right\}$$

$$A_\mu dx^\mu = a_0 dt + a_\varphi \sum_{k=1}^{N}\left(\prod_{l=0}^{k-1}\cos^2\theta_l\right)\sin^2\theta_k\varepsilon_k d\varphi_k$$

$\theta_0 \equiv 0,\ \theta_i \in [0,\pi/2],\ i = 1,\dots,N-1,\ \theta_N \equiv \pi/2,$
$\varphi_k \in [0,2\pi],\ k = 1,\dots,N,$ and $\varepsilon_k = \pm 1$

Introduction

•Only 4 metric functions for any odd D

•Other scalar fields added (e.g. dilaton Φ) will depend on r

•Exact solution exist only in few cases:
- 5D Einstein-Maxwell-Chern-Simons for $\lambda = 1$ (bosonic sector of minimal D=5 supergravity) (Cvetic et al (2004))
- Kaluza-Klein black holes (Einstein-Maxwell-dilaton)

•In pure EM theory of generic values of the coupling constants no solutions in closed form exist.

•Methods to approach the problem: numerical methods or perturbative ones.

•In this talk we will analyze a concrete theory (although the methods may be /have been applied to other theories): Einstein-Maxwell-dilaton (EMD) theory.

Einstein-Maxwell-Dilaton Black Holes

Einstein-Maxwell-Dilaton action

$$S = \int d^D x \sqrt{-g} \left(R - \frac{1}{2} \Phi_{,\rho} \Phi^{,\rho} - \frac{1}{4} e^{-2h\Phi} F_{\rho\sigma} F^{\rho\sigma} \right)$$

(units 16 π G_D=1) h=dilaton coupling constant

Field equations

$$G_{\rho\sigma} = \frac{1}{2} \left[\partial_\rho \Phi \partial_\sigma \Phi - \frac{1}{2} g_{\rho\sigma} \partial_\tau \Phi \partial^\tau \Phi + e^{-2h\Phi} \left(F_{\rho\tau} F_\sigma{}^\tau - \frac{1}{4} g_{\rho\sigma} F_{\tau\beta} F^{\tau\beta} \right) \right]$$

$$\nabla_\rho \left(e^{-2h\Phi} F^{\rho\sigma} \right) = 0$$

$$\nabla^2 \Phi = -\frac{h}{2} e^{-2h\Phi} F_{\rho\sigma} F^{\rho\sigma}$$

- Analytical solutions!!! Kaluza-Klein black holes
  Llatas 1997, Kunz et al. 2006
- h=0 corresponds to pure EM (no dilaton)

Einstein-Maxwell-Dilaton Black Holes

- Regular horizon at $r=r_H$ with $f(r_H)=0$
- Killing vector null at the horizon

$$\chi = \partial_t + \Omega \sum_{k=1}^{N} \varepsilon_k \partial_{\varphi_k}$$

Ω=horizon angular velocity

- The system of equations reduces to 6 second-order ODE's (a_0 may be eliminated by using a first integral associated to the charge)
- Generalized mass formula (valid also for non-Abelian BH's)

$$M = 2\frac{D-2}{D-3}\kappa A_H + \frac{D-2}{D-3}\sum_{i=1}^{N}\Omega_i J_i + 2\Psi_{el,H} Q + \frac{\Sigma}{h}$$

- We employed two methods: numerics and perturbations
- Results showed perfect agreement!!!
- Analytical KK solutions: a good test for the methods

$$h_{KK} = \frac{D-1}{\sqrt{2(D-1)(D-2)}}$$

Numerical method

- Equations solved with COLSYS (very high accuracy)
- Four degrees of freedom: {h, M, J, Q} so four numerical parameters {h, r_H, Ω, Q}
- Boundary conditions are crucial (Kunz et al. 2006)

$$f(r_H) = m(r_H) = n(r_H) = 0, \ \frac{\omega(r_H)}{r_H} = \Omega$$

$$a_{\varphi,r}(r_H) = 0, \ \Phi_{,r}(r_H) = 0$$

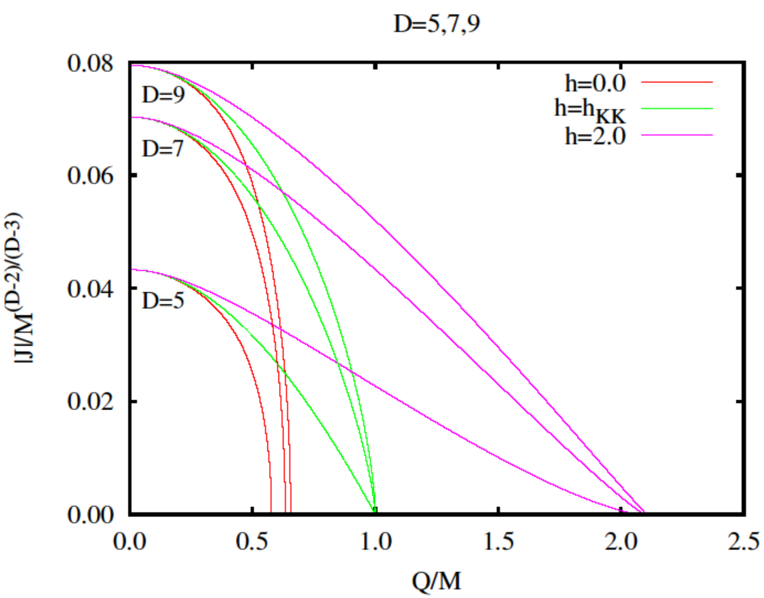
$$f(\infty) = m(\infty) = n(\infty) = 1, \ \omega(\infty) = 0$$

$$a_{\varphi}(\infty) = 0, \ \Phi(\infty) = 0$$

- Varying the numerical parameters we obtain the families of solutions

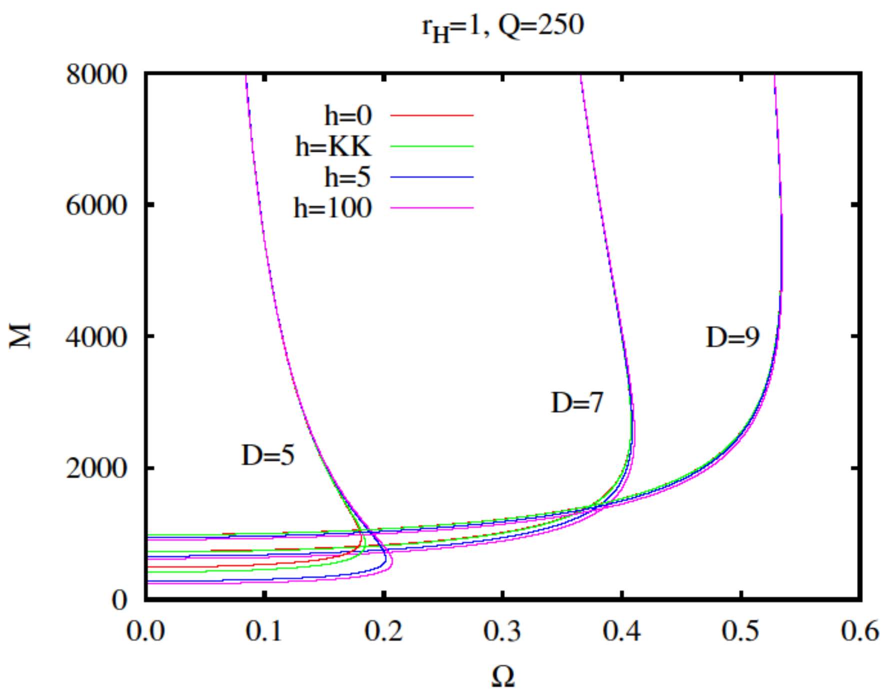
Numerical method

Domain of existence



Solutions are bounded by the extremal solutions beyond which naked singularities appear

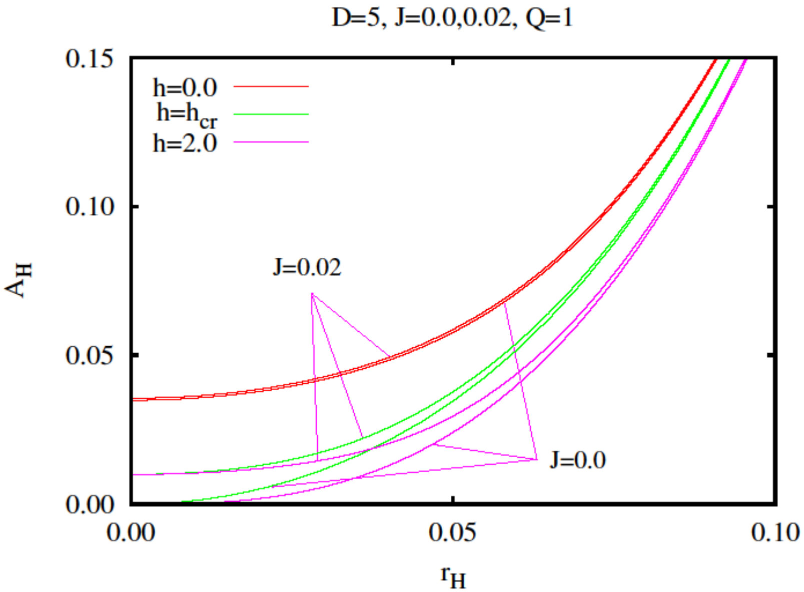
Variation of Ω

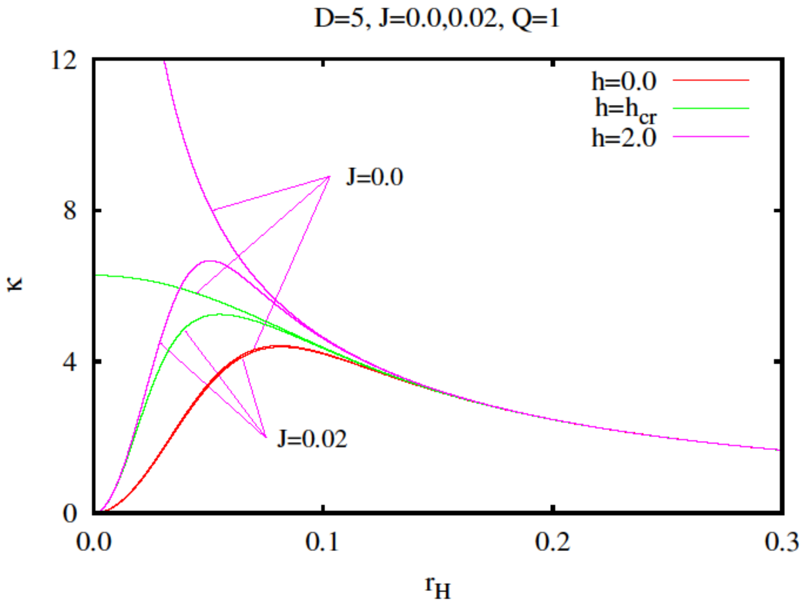


Similar behaviour as Kerr-Newman

Numerical method

- New features: Extremal limit (critical value) $h_{cr} = \frac{D-3}{\sqrt{2(D-2)}}$





static extremal black holes

$A_H = 0$ for $h > 0$

rotating extremal black holes

$A_H > 0$

static extremal black holes

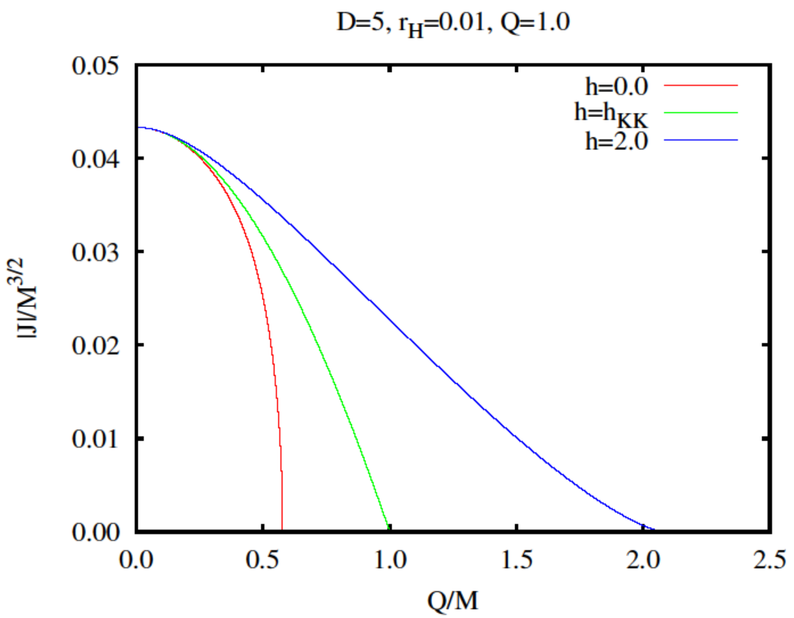
$\kappa \neq 0$ for $h_{cr} \leq h < \infty$

rotating extremal black holes

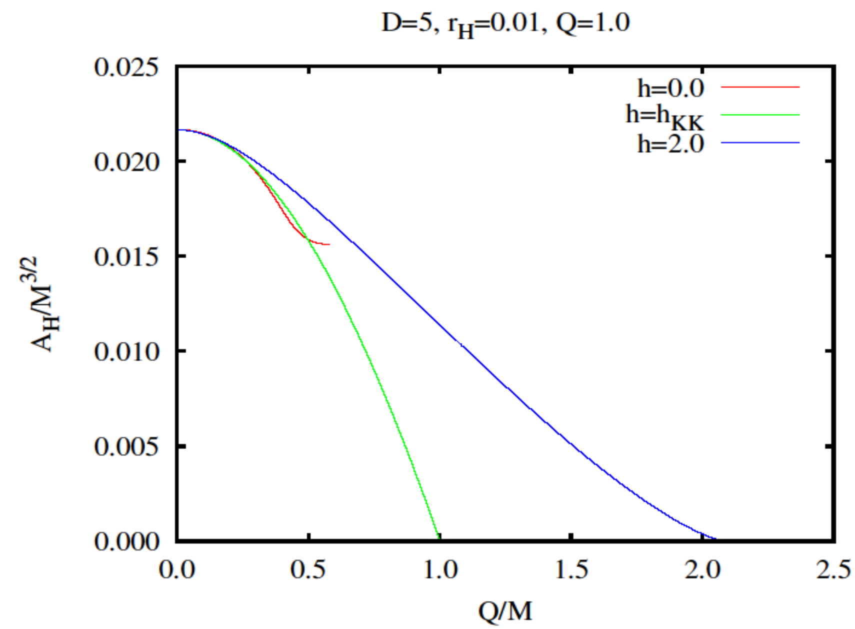
$\kappa = 0$

Numerical method

- Extremal limit



angular momentum

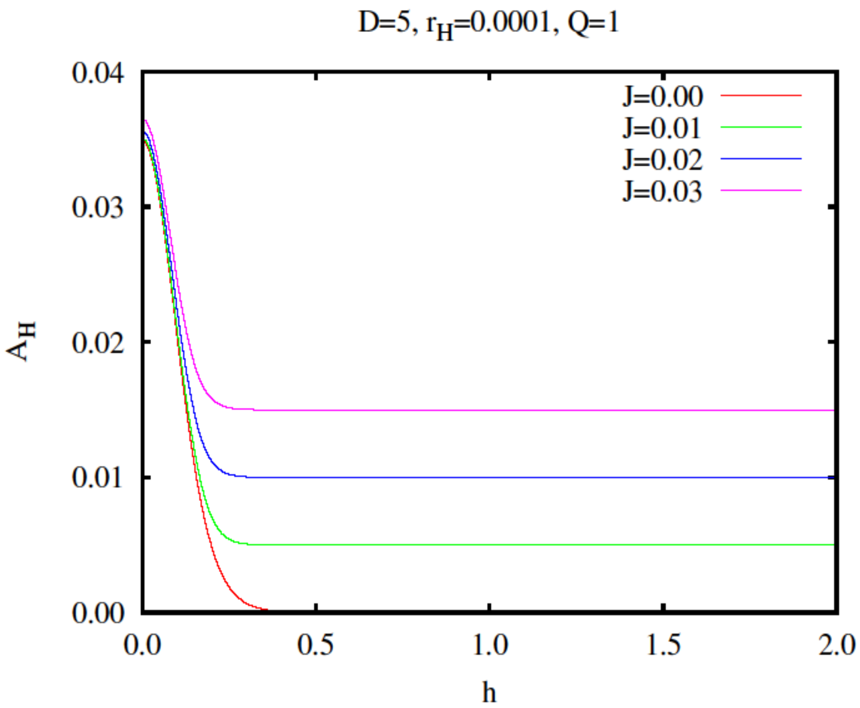


horizon area

Extremal EMD solutions: $J \sim A_H$, for $h \neq 0$

Numerical method

Area and angular momentum of (almost) extremal BH's



area vs dilaton coupling constant

conjecture for extremal black holes

odd D:

$$\frac{A_H}{J} = \frac{1}{\sqrt{2(D-3)}}$$

even D:

$$\frac{A_H}{J} = \frac{1}{2\sqrt{D-3}}$$

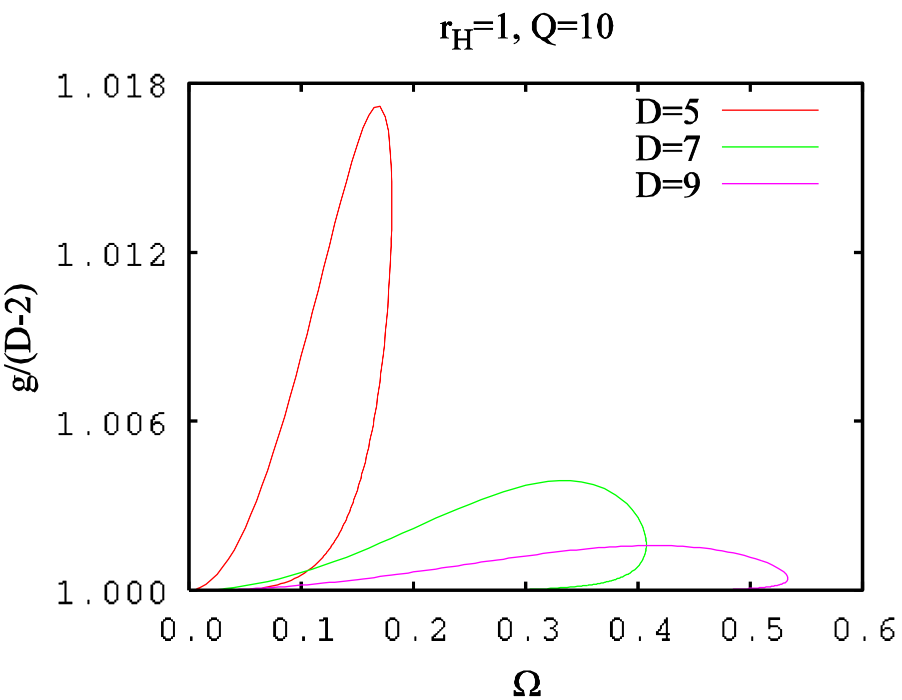
holds independent of Q (???)

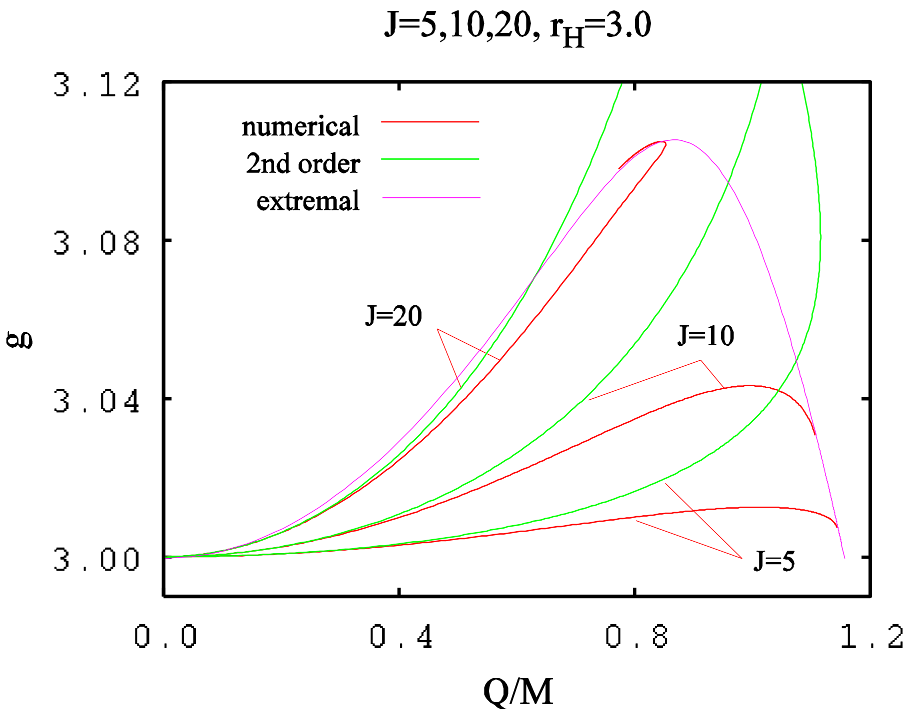
holds independent of h, except for $h = 0$

(holds for MP and KK)

Numerical method

- Pure EM solution (h=0)
- Gyromagnetic ratio $$g = \frac{2M\mu_{\mathrm{mag}}}{QJ}$$
- g=2 for D=4 but ...
- Perturbative value g=(D-2) (linear) Aliev 2006
- Numerical calculations: g is NOT constant for D>4 Kunz et al 2006





Perturbative method

- Typically one starts from an exact solutions and takes perturbations around it.
- In EMD theory there are several parameters to perform perturbations.
- Using rotation one normally is able to perform linear perturbations (Aliev 2006).
- But using the charge one may go beyond (Allaverdizadeh et al 2010).
- In fact, fastly rotating black holes (=extremal) are easier to perturb.
- We concentrate on perturbations of extremal solutions with the charge as the parameter .

Perturbative method

- Case with equal angular
- Boyer-Lindquist coordinates

$$
\begin{aligned}
ds^2 &= g_{tt}dt^2 + \frac{dr^2}{W} + r^2 \left[\sum_{i=1}^{N-1} \left(\prod_{j=0}^{i-1} \cos^2\theta_j \right) d\theta_i^2 + \sum_{k=1}^{N} \left(\prod_{l=0}^{k-1} \cos^2\theta_l \right) \sin^2\theta_k d\varphi_k^2 \right] \\
&+ V \left[\sum_{k=1}^{N} \left(\prod_{l=0}^{k-1} \cos^2\theta_l \right) \sin^2\theta_k \varepsilon_k d\varphi_k \right]^2 - 2B \sum_{k=1}^{N} \left(\prod_{l=0}^{k-1} \cos^2\theta_l \right) \sin^2\theta_k \varepsilon_k d\varphi_k dt \\
& A_\mu dx^\mu = a_t dt + a_\varphi \sum_{k=1}^{N} \left(\prod_{l=0}^{k-1} \cos^2\theta_l \right) \sin^2\theta_k \varepsilon_k d\varphi_k \\
& \Phi = \Phi(r)
\end{aligned}
$$

- Perturbations around the rotating uncharged solution: extremal Myers-Perry

Perturbative method

Perturbations

$$g_{tt} = -1 + \frac{2\hat{M}}{r^{D-3}} + q^2 g_{tt}^{(2)} + q^4 g_{tt}^{(4)} + O(q^6)$$

$$W = 1 - \frac{2\hat{M}}{r^{D-3}} + \frac{2\hat{J}^2}{\hat{M} r^{D-1}} + q^2 W^{(2)} + q^4 W^{(4)} + O(q^6)$$

$$V = \frac{2\hat{J}^2}{\hat{M} r^{D-3}} + q^2 V^{(2)} + q^4 V^{(4)} + O(q^6)$$

$$B = \frac{2\hat{J}}{r^{D-3}} + q^2 B^{(2)} + q^4 B^{(4)} + O(q^6)$$

$$\Phi = q^2 \Phi^{(2)} + q^4 \Phi^{(4)} + O(q^6)$$

$$a_t = q a_t^{(1)} + q^3 a_t^{(3)} + O(q^5)$$

$$a_\varphi = q a_\varphi^{(1)} + q^3 a_\varphi^{(3)} + O(q^5)$$

Perturbative method

•Extremal case is much simpler algebraically

$$\hat{M} = \frac{(D-1)^{\frac{(D-1)}{2}} \nu^{D-3}}{4(D-3)^{\frac{(D-3)}{2}}}, \qquad \hat{J} = \frac{(D-1)^{\frac{(D-1)}{2}} \nu^{D-2}}{4(D-3)^{\frac{(D-3)}{2}}}$$

•Perturbative equations must be solved order by order.

•Extremality and regularity of the solutions at the horizon are imposed. Also right asymptotic behaviour.

•Solutions to the equations contains algebraic functions of r, logarithms, and more complicated functions (the higher the order of the perturbation the more complicated the functions).

•One may extract the physical quantities from the solutions = perturbative analytical formulae!!!

•Comparison to analytically known solutions (KK) a good test for the perturbative scheme.

Perturbative method

$$M = A(S^{D-2})\left[\frac{\nu^{(D-3)}(D-2)(D-1)^{\frac{(D-1)}{2}}}{2(D-3)^{\frac{(D-3)}{2}}} + \frac{q^2(D-3)^{\frac{(D-1)}{2}}}{\nu^{D-3}(D-1)^{\frac{(D-1)}{2}}}\right] + O(q^4)$$

$$J = \frac{A(S^{D-2})\nu^{D-2}(D-1)^{\frac{(D-1)}{2}}}{(D-3)^{\frac{(D-3)}{2}}}$$

$$Q = A(S^{D-2})(D-3)q$$

$$\mu_{\text{mag}} = A(S^{D-2})(D-3)\left[q\nu - \frac{q^3(D-3)^{D-3}(D-3+2(D-2)h^2)}{\nu^{2D-7}(D-2)^2(D-1)^{D-2}}\right] + O(q^5)$$

$$g = D-2+q^2\frac{(D-3)^{D-3}((D-3)^2-2(D-1)(D-2)h^2)}{(D-2)(D-1)^{D-1}\nu^{2(D-3)}} + O(q^4)$$

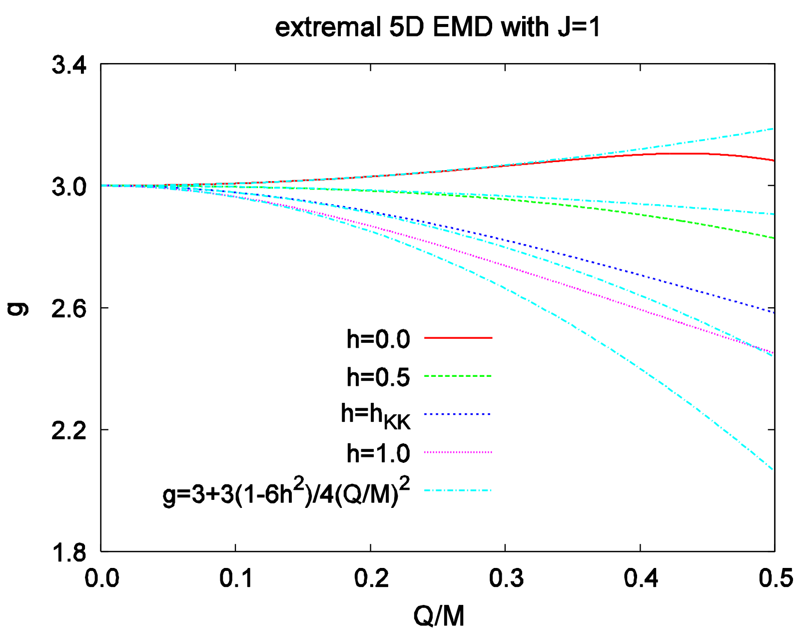
$$A_{\text{H}} = \frac{\sqrt{2}A(S^{D-2})(D-1)^{\frac{(D-1)}{2}}\nu^{D-2}}{2(D-3)^{\frac{(D-2)}{2}}} + O(q^4)$$

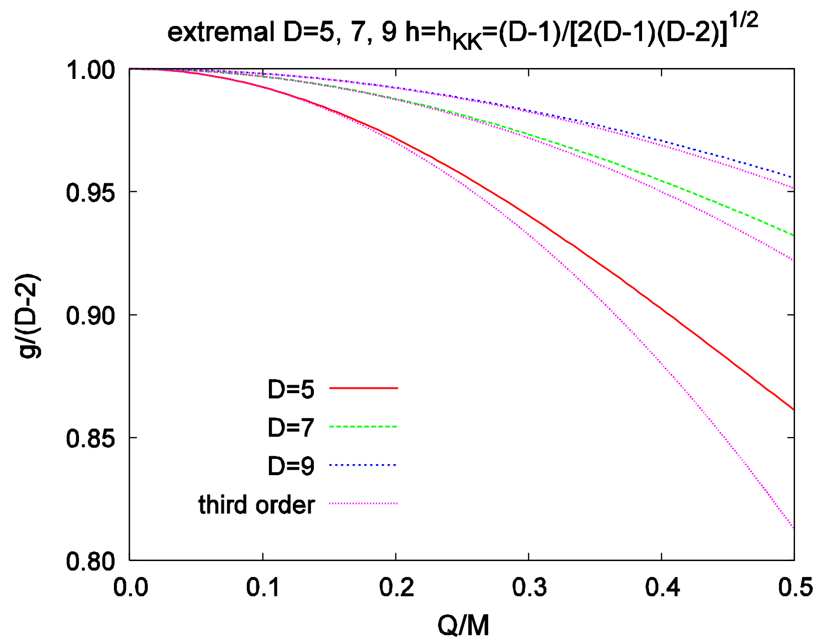
We confirm the numerical result for odd D:

$$\frac{A_H}{J} = \frac{1}{\sqrt{2(D-3)}}$$

Perturbative method

One can compare to numerics also





Perfect agreement in the range of validity of the perturbative scheme up that order

Conclusions

- Charged rotating higher dimensional black holes are difficult to obtain analytically except for several special situations.
- In odd dimensions when all angular momenta are equal the situation is much easier (even easier than in 4D!!!)
- Numerical and perturbative methods are of help to approach the problem.
- Both methods are complementary and in agreement.
- Black holes in higher dimensions possess new features still unexplored.