The parameter of the dark energy equation of state for high redshifts.

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Abstract

- We study the behaviour of the parameter $\omega(z)$ of the equation of state, $p = \omega(z)\rho$, as function of the redshift data from GRBs, to check its deviations from its most accepted value of -1.
- To this end we first find a reasonable calibration for the GRB in order to extract the luminosity distance d_L as a function of the redshift. Then we proceed to calculate the Hubble function $H(z)$ to obtain $\omega(z)$.

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2. $\omega(z)$ parameter of the dark energy EoS
3. Calibrating GRBs
4. Applying the calibrated relation to high redshifts
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The model with FRW spacetime

Our starting point are the well-known Friedmann equations for a homogeneous and isotropic universe,

$$H^2(a) = \left(\frac{\dot{a}}{a}\right)^2 = \sum_i \frac{8\pi G}{3}\rho_i(a), \qquad k = 0 \tag{1}$$

$$\frac{\ddot{a}}{a} = -\frac{4\pi G}{3}(\rho_i + 3p_i), \tag{2}$$

Here the diferent matter components labelled i, are all isotropic perfect fluids. We note from (2) that a component i can induce accelerated expansion provided $\rho_i + 3p_i < 0$.

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To fit with the supernovae redshift observations Eq. (1), demands another component besides the dust that models barionic matter, even if a non-zero curvature is assumed.

One possible solution to this puzzle is to modify the right hand side of the Friedmann equation introducing a new form of "dark" energy component, $\rho(a) = \rho_m(a) + \rho_X(a)$, with equation of state

$$\omega(z) = \frac{p_X(z)}{\rho_X(z)}. \tag{3}$$

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$\omega(z)$ parameter of the dark energy EoS

The parameter $\omega(z)$ determines not only the gravitational properties of dark energy but also its evolution. This evolution is easily obtained from the energy momentum conservation

$$d(\rho_X a^3) = -p_X d(a^3), \tag{4}$$

which leads to

$$\rho_X = \rho_{0X} e^{3\int_0^z \frac{dz'}{1+z'}(1+\omega(z'))}, \tag{5}$$

From previous equation we see that the determination of $\omega(z)$ is equivalent to that of $\rho_X(z)$ which in turn is related to the Hubble parameter $H(z)$, that from the first Friedmann equation and using (5) can be expressed as

$$H(z)^2 = H_0(z)^2[\Omega_{0m}(1+z)^3 + \Omega_{0X} e^{3\int_0^3 \frac{dz'}{1+z'}(1+\omega(z'))}]. \quad (6)$$

Thus, the knowledge of Ω_{0m}, z and $H(z)$ suffices to determine $\omega(z)$ which is obtained from the previous equation as

$$\omega(z) = \frac{\frac{2}{3}(1+z)\frac{d \ln H}{dz} - 1}{1 - \frac{H_0^2}{H^2}\Omega_{0m}(1+z)^3}. \quad (7)$$

From observational data it is possible to extract $d_L(z)$ and then determine $H(z)$, since,

$$H(z) = \left\{ \frac{d}{dz}\left(\frac{d_L(z)}{1+z}\right)\right\}^{-1}. \quad (8)$$

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Calibrating GRBs.

Based in the data of Type Ia Supernovae, Kodama et al. obtained the next calibration to the luminosity distance

$$\frac{d_L}{10^{27} cm} = 14{,}57 z^{1{,}02} + 7{,}16 z^{1{,}76}. \qquad (9)$$

With Eq. (9) and the 69 GRBs observational data, we can use the Eq. (8) and in this way infer if the dark energy effective EoS parameter ω is close to -1 (cosmological constant).

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However with the Kodama calibration assuming $H_0 = 70 kms^{-1} Mpc^{-1}$ and $\Omega_m = 0{,}32$, we obtained a plot for $w(z)$ that was physically unreasonable, diverging at certain redshift.

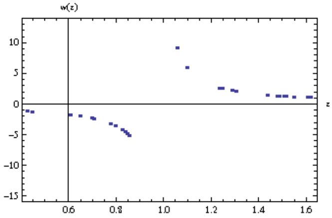

Figura: This figure shows how change the behavior of $w(z)$ for $z \approx 1$, which indicates a kind of "phase change" in the dark energy fluid.

We tried to repair this inconsistency by finding out another calibration for the GRBs extracted directly from Supernovae data.

We found the empirical formula of the luminosity distance of Type Ia supernovae from Riess et al. (2007) with $0{,}359 < z < 1{,}755$ as

$$\frac{d_L}{10^{27}cm} = 14{,}85z + 4{,}97z^2. \quad (10)$$

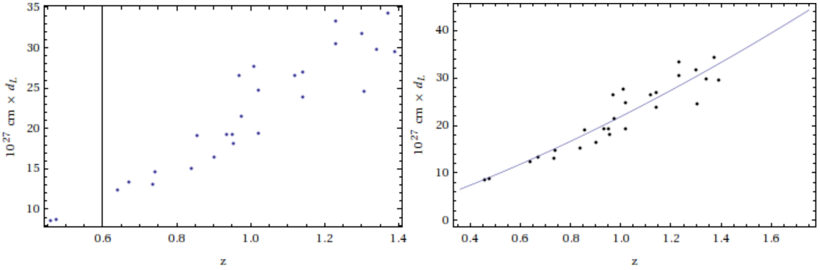

Figura: Luminosity distance as a function of the redshift of Type Ia supernovae.

We do not assume any cosmological models at this stage, but simply assume that the Type Ia supernovae are standard candles.

The typical spectrum of the prompt emission of GRBs can be expressed as exponentially connected broken power-law, so called Band function. Then we can determine spectral peak energy E_{peak}, corresponding to the photon energy at maximum in νF_ν spectra.

There are two empirical relations that relate prompt emission property with E_{peak}.

$E_{peak} - E_\gamma$ relation is the first one found by L. Amati et al., which connects E_{peak} with the isotropic equivalent energy E_γ.

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We apply the Eq. (10) to 16 GRBs with the redshift $z < 1{,}755$ in our sample to obtain $E_\gamma = 4\pi d_L^2 S_{bolo}(1+z)^{-1}F_{beam}$, where S_{bolo} is the bolometric fluence estimated in $1 - 10000$ keV energy range in GRB rest frame and F_{beam} is a beaming factor related with the jet opening angle, while $E_{peak} = (1+z)E_{p,obs}$.

In the figure we show the peak energy, E_{peak}, and the isotropic energy, E_γ, of 16 GRBs with $z < 1{,}755$. The solid line is the calibrated Amati relation given by,

$$\frac{E_\gamma}{10^{52} erg} = 3{,}41 \times 10^{-6} \left(\frac{E_{peak}}{1keV}\right)^{1{,}63}. \qquad (11)$$

The calibrated Amati relation.

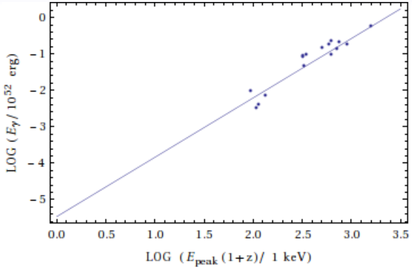

Figura: The peak energy (E_{peak}) and isotropic energy (E_γ) of 16 GRBs with $z < 1{,}755$. The solid line is the calibrated Amati relation given by Eq. (11).

Then, we apply this Amati relation to 26 GRBs with high redshifts, $z < 5{,}6$, to determine the luminosity distance as a function of z.

For each GRB with $z = z^i$ we have the observed S_{bolo} in the unit of erg cm^{-2}, the dimensionless F_{beam} and the observed $E_{p,obs}$ in the unit of keV. Then, using the equation 11 and $E_\gamma = 4\pi d_L^2 S_{bolo}(1+z)^{-1} F_{beam}$, the luminosity distance can be derived as

$$d_L(z^i) = 10^{23} cm \sqrt{\frac{3,41}{4\pi F^i_{beam} S^i_{bolo}}} (E^i_{p,obs})^{0,815} (1+z^i)^{1,315}. \qquad (12)$$

From here, usually it is tested a cosmological model

$$\Delta\chi^2 = \sum_i \left(\frac{log d_L(z^i) - log d_L^{th}(z^i, \Omega_m, \Omega_\Lambda)}{\Delta d_L(z^i)} \right)^2 - \chi^2_{best}, \qquad (13)$$

where χ^2_{best} means the chi-square value for the best fit parameter set of Ω_m and Ω_Λ.

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With the Eq. (12) and the data of GRBs, we obtained the next plot for the luminosity distance with $0{,}359 < z < 5{,}6$,

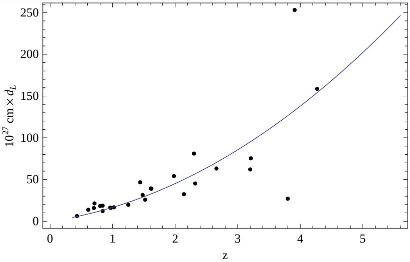

Figura: $d_L(z)$ from the luminosity distance derived with the Amati relation.

and the empirical formula for the luminosity distance is

$$\frac{d_L}{10^{27}cm} = 10{,}68z + 5{,}95z^2 \tag{14}$$

Using this extracted rule for the luminosity distance as function of z, Eq. (14) and using Eq. (7), we derived the dark-energy equation-of-state parameter $w(z)$

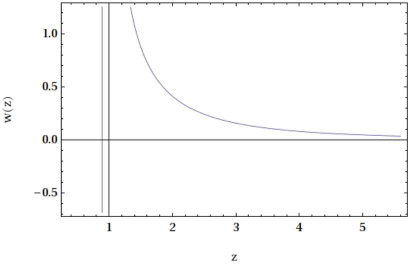

Figura: $w(z)$ parameter of the equation of state as a function of z, derived from 26 subset of GRBs given in Schaefer and $H_0 = 85{,}12 kms^{-1} Mpc^{-1}$, $\Omega_{0m} = 0{,}32$.

The blows up of $\omega(z)$

$$\omega(z) = \frac{\frac{2}{3}(1+z)\frac{d\ln H}{dz} - 1}{1 - \frac{H_0^2}{H^2}\Omega_{0m}(1+z)^3}, \quad (15)$$

occur when $H^2(z) = H_0^2 \Omega_{0m}(1+z)^3$.

Note also that we use $H_0 = 85{,}12 kms^{-1} Mpc^{-1}$ because this value is predicted by the Eqs. (8) and (14),

$$H(z) = \left\{ \frac{d}{dz}\left(\frac{d_L(z)}{1+z}\right)\right\}^{-1} = \frac{(1+z)^2}{0{,}012 + 0{,}013z + 0{,}007z^2}. \quad (16)$$

So,

$$H(z=0) \equiv H_0 = 85{,}12 kms^{-1} \mathrm{Mpc}^{-1}. \quad (17)$$

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On the other hand, the deceleration parameter $q(z)$ as function of z is

$$-q(z) = 1 - (1+z)\frac{H'(z)}{H(z)} = \frac{0{,}007(0{,}108+z)(1{,}892+z)}{0{,}012+0{,}013z+0{,}007z^2}, \quad (18)$$

and at $z=0$, we have that $q(z=0) = -0{,}114$, in other words, the universe is in acelerated expansion.

Conclusions

- The aim of this work is to determine the functional dependence of the dark-energy equation-of-state parameter in terms of the redshift, $w(z)$, from observational data coming from the GRBs.
- First we find the best calibration between the observational data of GRBs, obtaining the luminosity distance as function of redshift, $d_L(z)$, Eq. (14). Then we obtain the corresponding Hubble function and then $w(z)$.

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- The estimations for $H_0 = 85{,}12 \mathrm{km} s^{-1} \mathrm{Mpc}^{-1}$ and $q(z=0) = -0{,}114$ have the correct order of magnitude.
- To keep $\omega(z)$ finite, we have to use a very small value for Ω_{0m}.
- GRBs data by themselves are unable to strongly to constraint cosmological parameters. GRBs should be used combined with other data sets.
- Qualitatively our results are in agreement with related works by Kodama et al. (2009), Capozziello and Izzo (2009) and Samushia and Ratra1 (2010).

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