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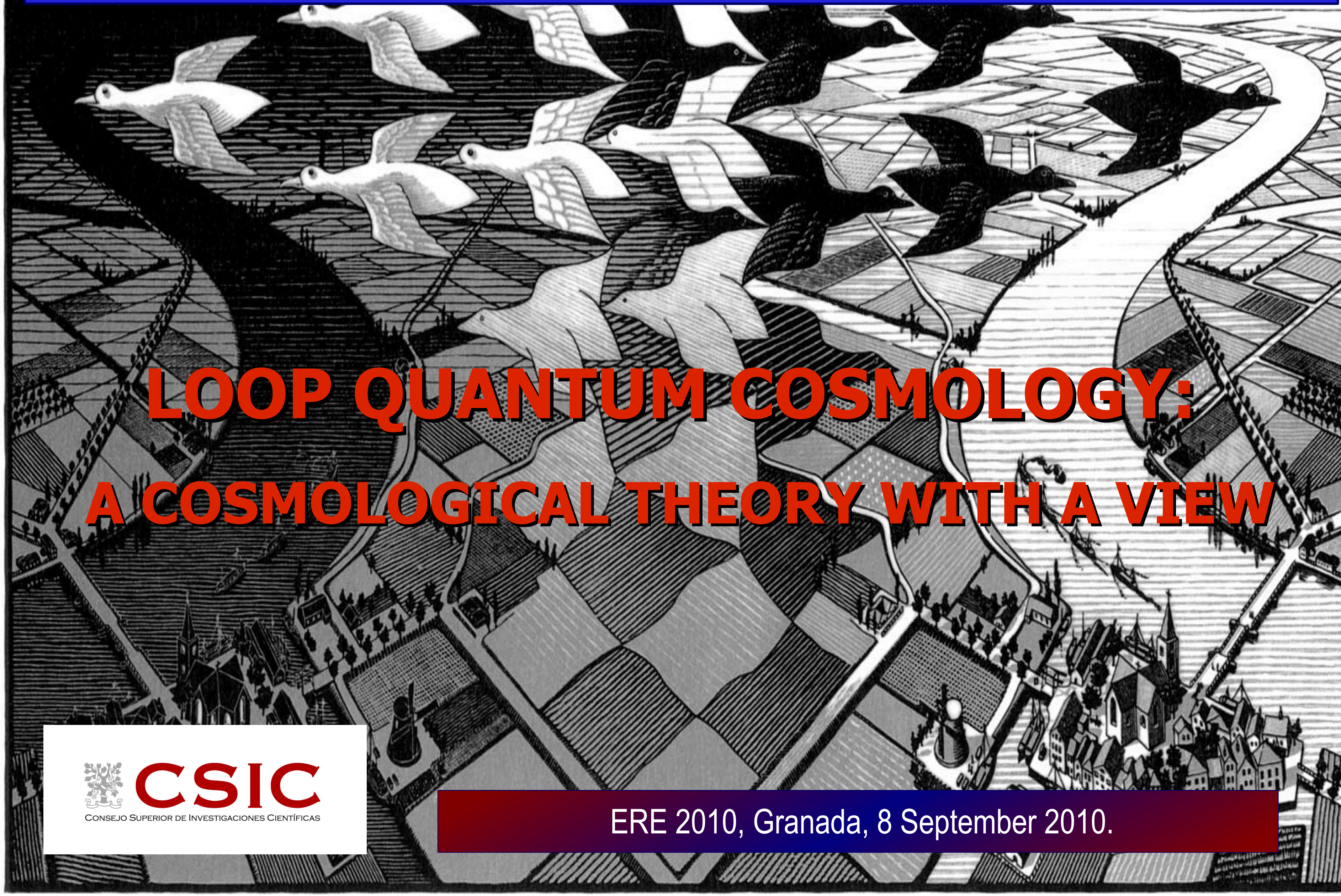

LOOP QUANTUM COSMOLOGY:

A COSMOLOGICAL THEORY WITH A VIEW

CSIC
Consejo Superior de Investigaciones Científicas

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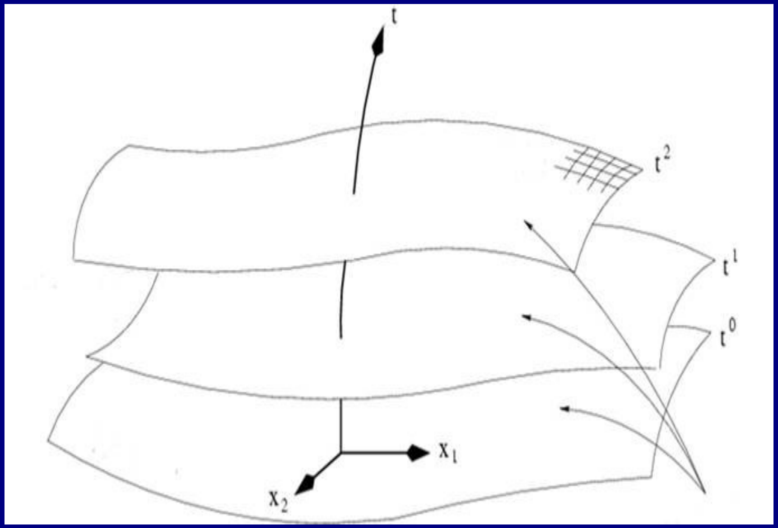
Introduction

- Do we need a **quantum theory of gravity**? Gravity is the only fundamental physical interaction not (satisfactorily) quantized yet.
- General Relativity (GR) leads to **singularities** where predictability breaks down.
- Any quantum theory of gravity must have the infrared behavior of GR. Explaining why the Universe is so classical is a challenge.
- Quantum phenomena may open a window to new physics.
- Geometrodynamics in the Wheeler-De Witt approach is not a valid answer.
- Loop Quantum Gravity (LQG) is a **nonperturbative,** diffeomorphism invariant and **background independent** quantum theory of gravity.
- Its application to simple cosmological models gives rise to Loop Quantum Cosmology (LQC).

Canonical GR

- Consider globally hyperbolic spacetimes. In geometrodynamics, the inital data are contained in the **induced 3-metric** and the **extrinsic curvature**.

  We can introduce triads to allow coupling with fermions. A canonical set is formed by the **densitized triad** and the triadic **extrinsic curvature**.



$$E^a_i = \sqrt{\det h}\; e^a_i , \quad K^i_a$$

- We can replace the extrinsic curvature by a **connection** valued 1-form which takes values on **su(2)**.

Classically, this Ashtekar-Barbero connection is $A^i_a = \Gamma^i_a + \gamma K^i_a$.

γ is the Immirzi parameter.

Γ^i_a is the su(2)-connection compatible with the co-triad.

A canonical set is $$A^i_a = \Gamma^i_a + \gamma K^i_a , \quad E^a_i .$$ (modulo $8\pi G\gamma$)

Holonomy-flux algebra

- The gauge invariant information about the connection is captured by the Wilson loops. We replace the connection by SU(2)-holonomies along (piecewise analytic) edges.

$$h_e = P \exp \int_e A_a^i \tau_i dx^a .$$

This involves a 1-dimensional smearing. $2i\tau_j = \sigma_j$ are the Pauli matrices.

- We also smear the vector density E_i^a without introducing any background dependence. This leads naturally to FLUXES through surfaces.

$$E(S, f) = \int_S E_i^a f^i \epsilon_{abc} dx^b dx^c .$$

- Holonomies and fluxes form an algebra under Poisson brackets. Quantization requires the representation of this algebra.

- Gravity is a constrained system: Gauss [SU(2)], Diffeomorphisms, and Hamiltonian or scalar constraints.

LOST theorem

- There exists a **UNIQUE** cyclic representation of the holonomy-flux algebra with diffeomorphism-invariant ("vacuum") state.

- We call **cylindrical** the functions that depend on the connection only via the holonomies along a finite number of edges (forming a graph).

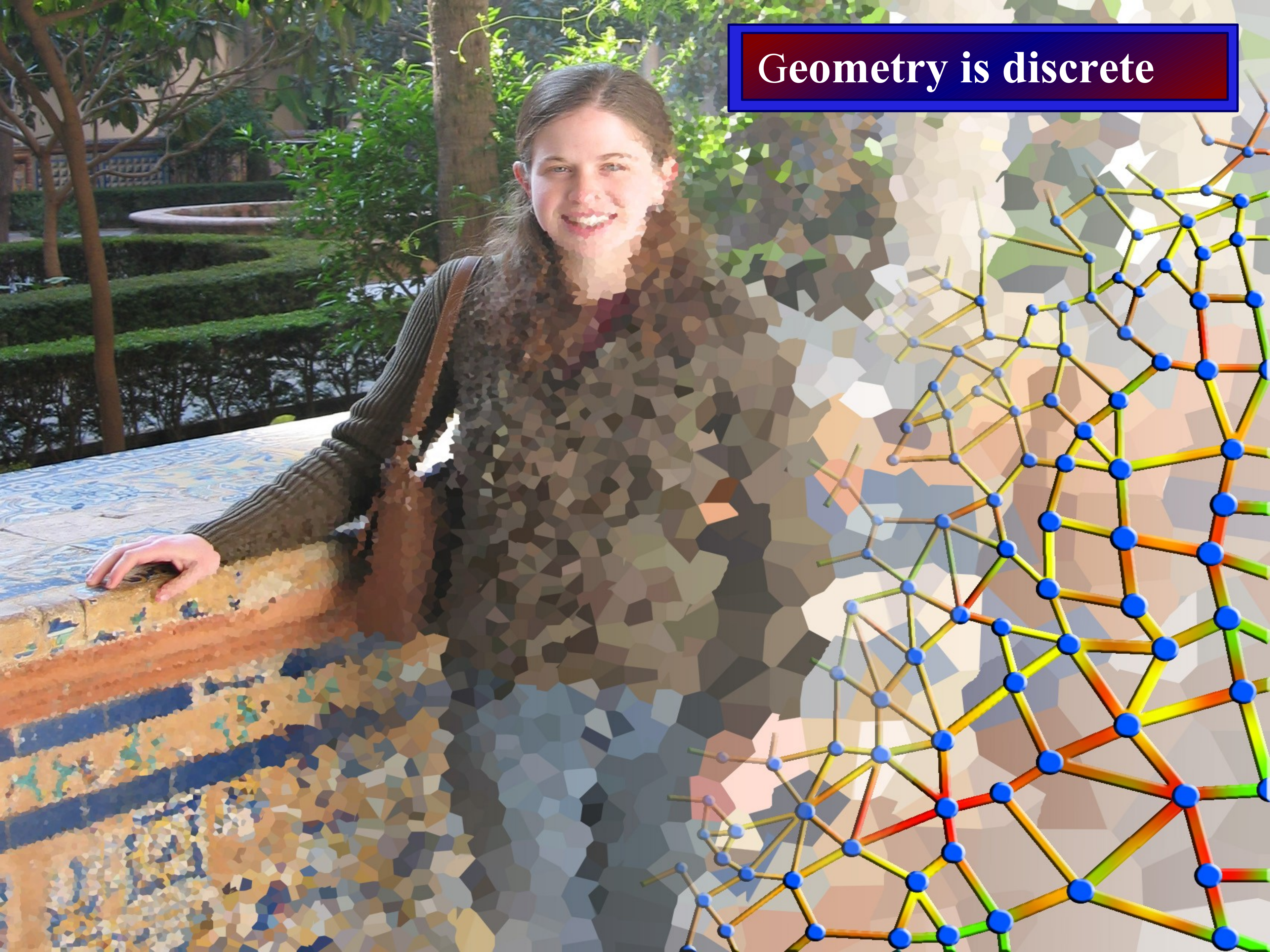
  Completing this algebra of functions with respect to the norm of the supremum, we obtain a commutative C*-algebra with identity.

- This algebra is (isomorphic to) the algebra of continuous functions on a **compact** space, called the **spectrum**. Smooth connections are *dense*.

- The Hilbert space of the representation is that of square integrable functions on the spectrum with respect to a **diffeomorphism-invariant** measure: The Ashtekar-Lewandowski measure.

- The connection cannot be defined as an operator valued distribution.

  The representation is **NOT EQUIVALENT** to a standard one.


Geometry is discrete

LQC: Flat FRW model

- Let us apply these quantization techniques to **homogeneous and isotropic flat** (FRW) **cosmologies**. We include a massless minimally coupled scalar field.

- We introduce a **fiducial triad** ${}^0e^a_i$, and an integration **cell** adapted to it. We call V_0 its fiducial volume.

- Physical results are independent of these choices.

- One can fix the gauge and diffeomorphism freedom so that

$$A_a = c\, V_0^{-1/3}\, {}^0e^i_a \tau_i; \qquad E^a = p\, V_0^{-2/3} \sqrt{\det {}^0h}\ \ {}^0e^a_i \tau^i .$$

- The variables (c, p) describe the geometry degrees of freedom and are canonical.

$$\{c, p\} = \frac{8\pi\gamma G}{3}.$$

- Their (classical) relation with the scale factor is $c = \gamma V_0^{1/3} \dot{a}, \quad |p| = V_0^{2/3} a^2 .$

LQC: Holonomies, fluxes and Bohr compactification

- It suffices to consider holonomies along (fiducial) **straight edges**.

$$h_{{}^0 e_i}(\lambda) = \cos\left(\frac{\lambda c}{2}\right)\mathbf{1} + 2\sin\left(\frac{\lambda c}{2}\right)\tau_i .$$

Triads are now smeared accross squares. Fluxes are totally determined by p.

- The configuration algebra is the linear space of **continuous** and **bounded** complex functions in $\mathbb{R}$.

$$f(c) = \sum_j f_j \, e^{i\lambda_j c} .$$

Its completion is the Bohr C*-algebra of almost periodic functions.
Its spectrum is the Bohr compactification of the real line, $\mathbb{R}_{Bohr}$.

- This compactification can be seen as the set of group homomorphisms from the group $\mathbb{R}$ (with the sum) to the multiplicative group T of **unitaries** in $\mathbb{C}$.

- The real line is **dense.**

$$\forall c \in \mathbb{R} \quad x_c : \mathbb{R} \to T ; \quad x_c(\lambda) = e^{i\lambda c} .$$

LQC: Momentum representation

- Since the group $\mathbb{R}_{Bohr}$ is compact, it has an invariant **Haar measure**.

- The functions on $\mathbb{R}_{Bohr}$ consisting in the EVALUATION at a real point μ form an **orthonormal basis.** We designate each element with the ket $|\mu\rangle$.

$$\forall \mu_1, \mu_2 \in \mathbb{R}, \ \langle \mu_1 | \mu_2 \rangle = \delta^{\mu_1}_{\mu_2}.$$

- Calling $N_\lambda := e^{i\lambda c/2}$, the "momentum" representation is given by

$$\hat{p}|\mu\rangle = \frac{4\pi\gamma G}{3}\mu|\mu\rangle, \qquad \hat{N}_\lambda|\mu\rangle = |\mu+\lambda\rangle.$$

- The Hilbert space is **nonseparable**, but states differ from zero only on a countable subset of the real line.

$$|\psi\rangle := \sum_{\mu\in\mathbb{R}} \psi(\mu)|\mu\rangle; \quad \sum_{\mu\in\mathbb{R}} |\psi(\mu)|^2 < \infty.$$

- The representation **fails to be continuous.** The connection operator does not exist. The representation is **INEQUIVALENT** to the Wheeler-De Witt one.

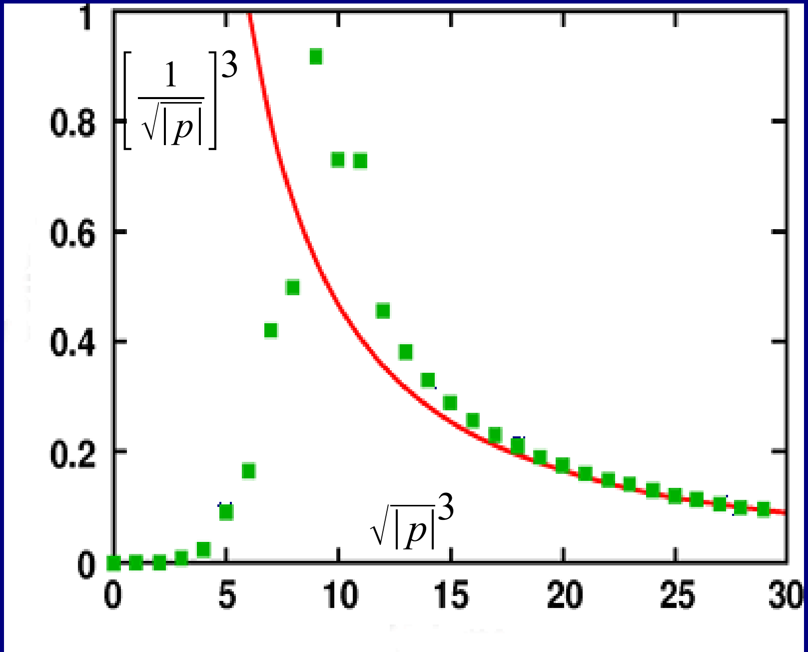
LQC: Triad operator

- The **triad** diverges at the Big-Bang, $e_i^a = sign(p)|p|^{-1/2} V_0^{1/3} \; {}^0e_i^a .$

- Since $\hat{p}$ has a **point** real spectrum, $|\hat{p}|^{-1}$ is not well defined.
But it is possible to define a triad in terms of elementary operators.

- Classically, $\frac{sign(p)}{\sqrt{|p|}} = \frac{4}{\bar{\mu}} trace\left(\sum_i \tau^i h_{{}^0e_i}(\bar{\mu})\{h^{-1}_{{}^0e_i}(\bar{\mu}), \sqrt{|p|}\}\right).$

$$\frac{\bar{\mu}}{6}\left[\widehat{\frac{sign(p)}{\sqrt{|p|}}}\right] = \hat{N}_{-\bar{\mu}}|\hat{p}|^{1/2}\hat{N}_{\bar{\mu}} - \hat{N}_{\bar{\mu}}|\hat{p}|^{1/2}\hat{N}_{-\bar{\mu}} .$$



- The operator is diagonal in this basis.
- It is bounded from above.
- The classical divergence disappears.

LQC: Curvature operator

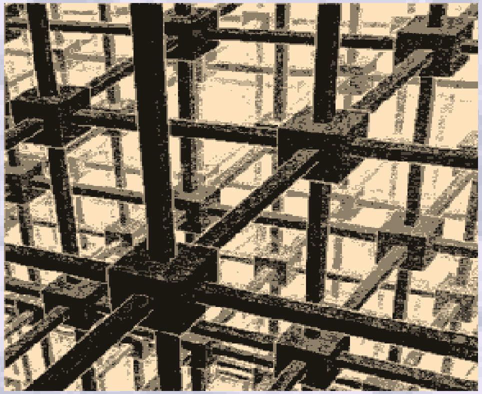
- We define the **curvature** operator using a square **loop of holonomies**:

$$F^k_{ab} = -2 \lim_{\bar\mu} trace\left(\frac{h_{[ij]}(\bar\mu) - 1}{\bar\mu^2 V_0^{2/3}} \tau^k \, {}^0e^i_a \, {}^0e^j_b \right), \quad h_{[ij]}(\bar\mu) := h_{{}^0e_i}(\bar\mu) h_{{}^0e_j}(\bar\mu) h^{-1}_{{}^0e_i}(\bar\mu) h^{-1}_{{}^0e_j}(\bar\mu).$$

One cannot take the limit of zero **regulator** $\bar\mu$. This introduces a **nonlocality**.

- In LQG, the **area spectrum** has a minimum non-zero eigenvalue Δ. The regulator is fixed so that the physical area of the square coincides with this area gap. It is **state-dependent**.

$$\bar\mu^2 |p| = \Delta .$$



- We relabel the $|\mu\rangle$-basis as a basis of **volume** eigenstates, introducing an affine parameter for the "translations" generated by $(\bar\mu c)/2$:

$$v = [2\pi\gamma G \sqrt{\Delta}]^{-1} sign(p) \; |p|^{3/2} \quad \longrightarrow \quad \hat N_{\bar\mu} |v\rangle := |v+1\rangle .$$

The **physical volume** of the fiducial cell is $V = |p|^{3/2}$.

LQC: Hamiltonian constraint

- We use the **standard** Schrödinger **representation** for the matter field ϕ. The Hilbert space is the tensor product of the polymeric one and $L^2(\mathbb{R}, d\phi)$.

- One gets the Hamiltonian constraint (with a suitable factor ordering):

$$\hat{H} := -6\,\widehat{\Omega}^2 + \hat{P}_\phi^2;$$

$$\widehat{\Omega} := \frac{-i}{8\gamma\sqrt{2\pi G\Delta}}\left[\widehat{\frac{1}{\sqrt{|p|}}}\right]^{-1/2}\widehat{\sqrt{|p|}}\left[\left(\hat{N}_{2\bar{\mu}}-\hat{N}_{-2\bar{\mu}}\right)\widehat{sign(p)} + \leftarrow(\mathbf{sym})\rightarrow\right]\widehat{\sqrt{|p|}}\left[\widehat{\frac{1}{\sqrt{|p|}}}\right]^{-1/2}$$

- This constraint leaves invariant the zero-volume state and its orthogonal complement. The classical **singularity** is **removed from the Hilbert space!**

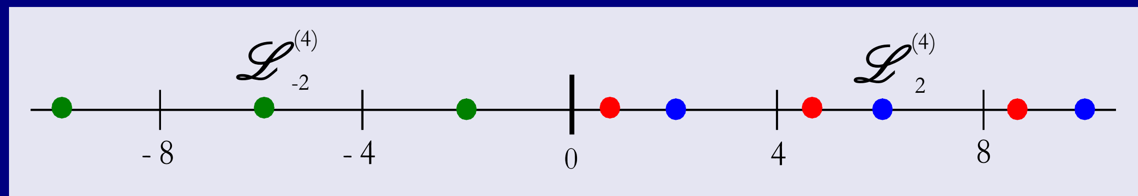
- The action of $\widehat{\Omega}^2$ is

$$\widehat{\Omega}^2|v\rangle = f_+(v)|v+4\rangle + f(v)|v\rangle + f_-(v)|v-4\rangle,$$

where the real function $f_+(v)$ $\left(f_-(v)\right)$ has the remarkable property that it vanishes in the **whole interval** $[-4,0]$ $\left([0,4]\right)$.

LQC: Superselection and states

- $\widehat{\Omega}^2$ preserves the semilattices $\mathscr{L}^{(4)}_{\pm\epsilon} := \{\pm(\epsilon + 4n),\ \ n \in \mathbb{N}\}$ with ϵ in (0,4].



- Each semilattice provides a **superselection sector.**
  On it, $\widehat{\Omega}^2$ has a nondegenerate absolutely continous spectrum equal to $\mathbb{R}^+$.

- $\widehat{\Omega}^2$ can be seen as a second-order difference operator. But its eigenfunctions are entirely determined by their value at ϵ. It is a **No-Boundary** prescription.

- These **eigenfunctions** $e^{\epsilon}_{\delta}(v)$ are **REAL** (up to a global phase).

- **Solutions** to the constraint $\hat{P}^2_{\phi} = 6\,\widehat{\Omega}^2$ have the form

$$\psi(v,\phi) = \int_0^{\infty} d\delta\, e^{\epsilon}_{\delta}(v)\left[\psi_+(\delta)e^{i\sqrt{6}\delta\phi} + \psi_-(\delta)e^{-i\sqrt{6}\delta\phi}\right].$$

Physical states are (e.g.) positive frequency solutions in $L^2(\mathbb{R}^+, d\delta)$.

LQC: Big Bounce

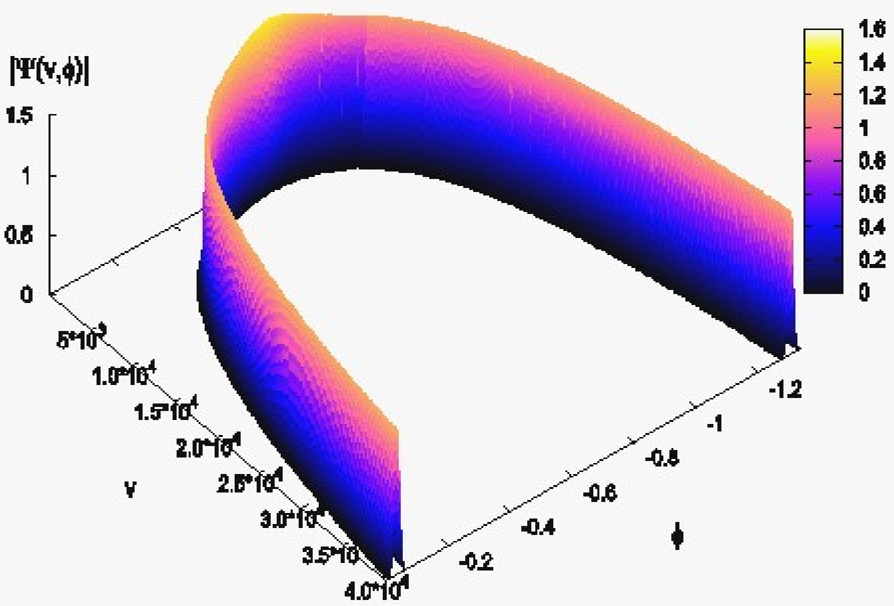
- A complete set of Dirac observables is given by $\hat{P}_\phi$ and $|\hat{v}|_{\phi_0}$, the latter being defined by the action of $|\hat{v}|$ when the field is $\phi = \phi_0$.

- Consider (Gaussian-like) positive frequency states **peaked** at certain values $P_\phi = P_\phi^0$ and $v = v^0$ for a fixed $\phi = \phi_0 \gg 1$.

We study "semiclassical" states with

$$|v^0|, |P_\phi^0| \gg 1 .$$

(Ashtekar, Pawlowski & Singh)



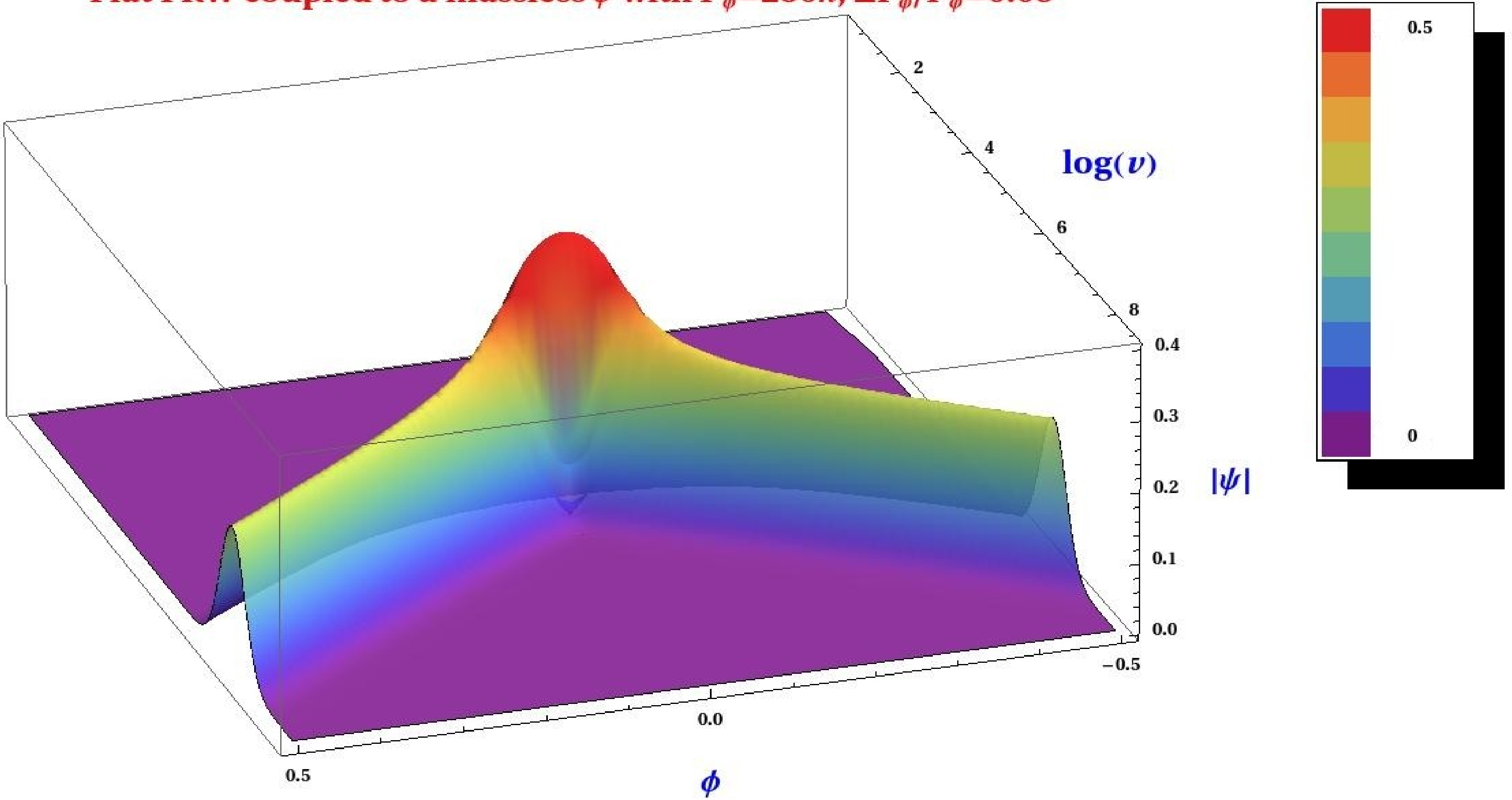
- The Big Bang is replaced by a quantum bounce:

  The Big Bounce

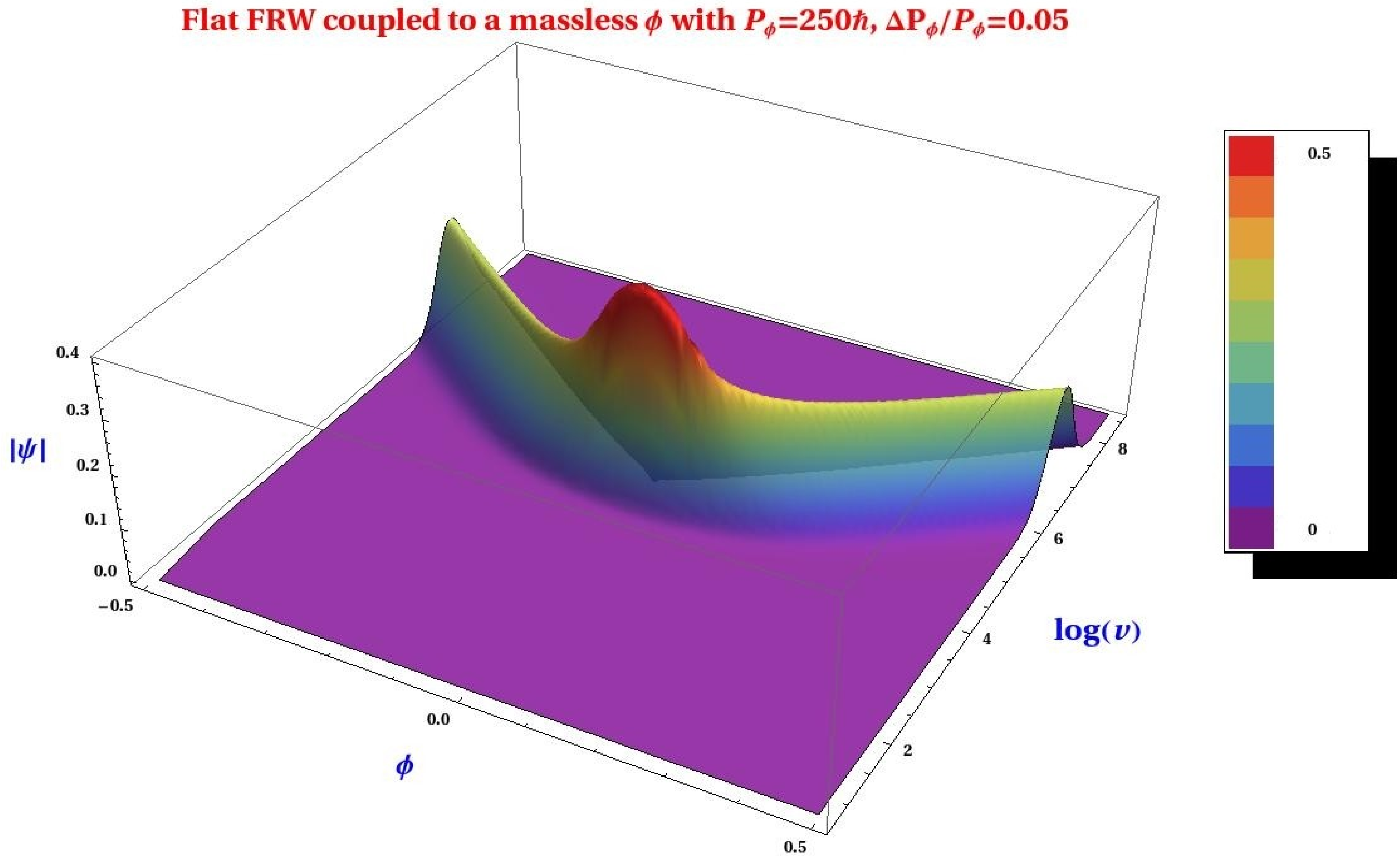
LQC: Big Bounce

(Martín-Benito, Mena Marugán & Olmedo)

Flat FRW coupled to a massless ϕ with P_ϕ=250$\hbar$, $\Delta P_\phi/P_\phi$=0.05



LQC: Big Bounce



LQC: Big Bounce

- Semiclassical states remain **peaked**.
- The trajectory deviates from GR when $\rho > 0.01\,\rho_{crit}$.
  The scale for the onset of corrections is universal:

$$\rho_{crit} = \frac{\sqrt{3}}{32\pi^2\gamma^3 G^2} \approx 0.41\,\rho_{Planck}.$$

- In this regime, gravity behaves as a **repulsive** force.
- The matter density is BOUNDED by ρ_{crit} in the trajectories of the peaks.
  This bound coincides with the **supremum** of the density operator.
- The trajectory matches an EFFECTIVE DYNAMICS, derived with techniques of geometric quantum mechanics (Taveras).
- The volume at the bounce scales with P_ϕ and can be as large as desired.
  It does not control the emergence of quantum phenomena.

Generality of the bounce

The eigenfunctions of the gravitational constraint are REAL:
The ingoing and outgoing components have equal amplitude.

(Martín-Benito, Mena Marugán & Olmedo. Also Ashtekar, Corichi & Singh for one superselection sector)

LQC: Generality of the bounce

- Using the **effective** equations, one can show that all **strong** singularities (à la Królak) are RESOLVED in flat FRW for any kind of matter content. Only Type II and Type IV singularities may remain (Singh).

- Similar results about the bounce in other isotropic models:

  - **Flat** FRW with **negative cosmological constant** (Bentivegna & Pawlowski).
  - **Closed** FRW (Ashtekar, Pawlowski, Singh, Vandersloot...).

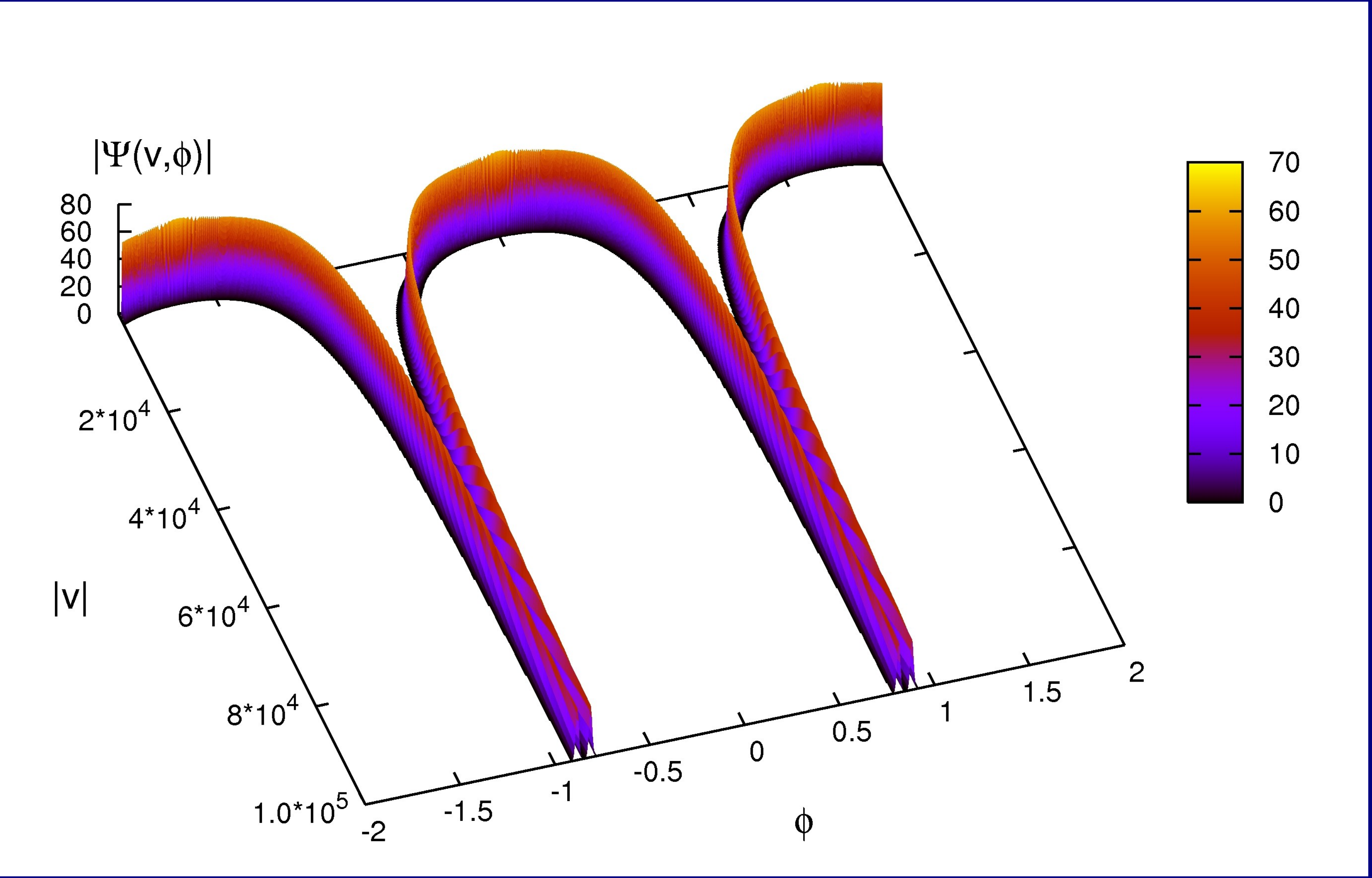
  } Also Big Crunch, like in GR.

  - **Open** FRW (Vandersloot. Ashtekar & Wilson-Ewing).
  - Flat FRW with positive cosmological constant (Ashtekar & Pawlowski): Solutions reach infinity at finite emergent time ϕ and can be continued. The constraint has different self-adjoint extensions (same physics). There is an upper bound for the cosmological constant.

- The critical density acquires the same value.

LQC: Generality of the bounce

- Flat FRW with positive cosmological constant (Pawlowski).



LQC: Anisotropic models

- LQC has been implemented successfully in anisotropic models:
  - Bianchi II (Ashtekar & Wilson-Ewing).
  - Bianchi IX (Wilson-Ewing).
  - Bianchi I (Chiou. Martín-Benito, Mena Marugán & Pawlowski. Ashtekar & Wilson-Ewing...).

- **Numerical** simulations confirm the "Bounce". Together with the **BKL conjecture**, this suggests a generic resolution of spacelike singularities.

- Bianchi I has been studied with an **improved** dynamics motivated from LQG:
  - The "basic" holonomies produce a CONSTANT shift in the volume.
  - Data on the first section of constant volume determine the solution. This allows one to solve the constraint and get the Hilbert space of physical states. This is so even in VACUO (M-B, MM, W-E).
  - The superselection sectors for the anisotropies are discrete, but DENSE. Different triad orientations are not mixed (M-B, MM, W-E).

LQC: Effective analyses

- The effective equations for the improved dynamics in Bianchi I guarantee that the directional Hubble rates $(H_I = \dot{a}_I / a_I)$, the expansion and the shear scalar (of comoving observers) are bounded from above (Corichi & Singh).

- Occasionally effective equations have been misused. Wrong statements are:

- *In flat FRW, if the evolution leads to a vanishing or a divergent value of the scale factor then the universe is asymptotically de Sitter in that regime* (Singh).
  The mistake comes from the fact that the convergence of $\int^{\infty} [1+w(a)] da/a$ does not require $w(a) \rightarrow -1$. Also, one can have $\rho(a=0)=0$ (David Jaramillo).

- *Since the [effective] phase space function that represents expansion [or shear] is bounded [unbounded], the corresponding operator in the quantum theory will have a bounded [unbounded] spectrum* (Corichi & Singh).
  This is not true for functions that depend on noncommuting variables.

- *In an effective treatment [including all quantum moments], we can ... focus on the algebra of observables. Results will thus be manifestly representation independent* (Bojowald & Tsobanjan).
  This involves assumptions that are not satisfied, e.g., with superselection.

LQC: Superinflation

- Consider **flat FRW with an inflaton** field whose kinetic energy is positive. The EFFECTIVE equations imply:

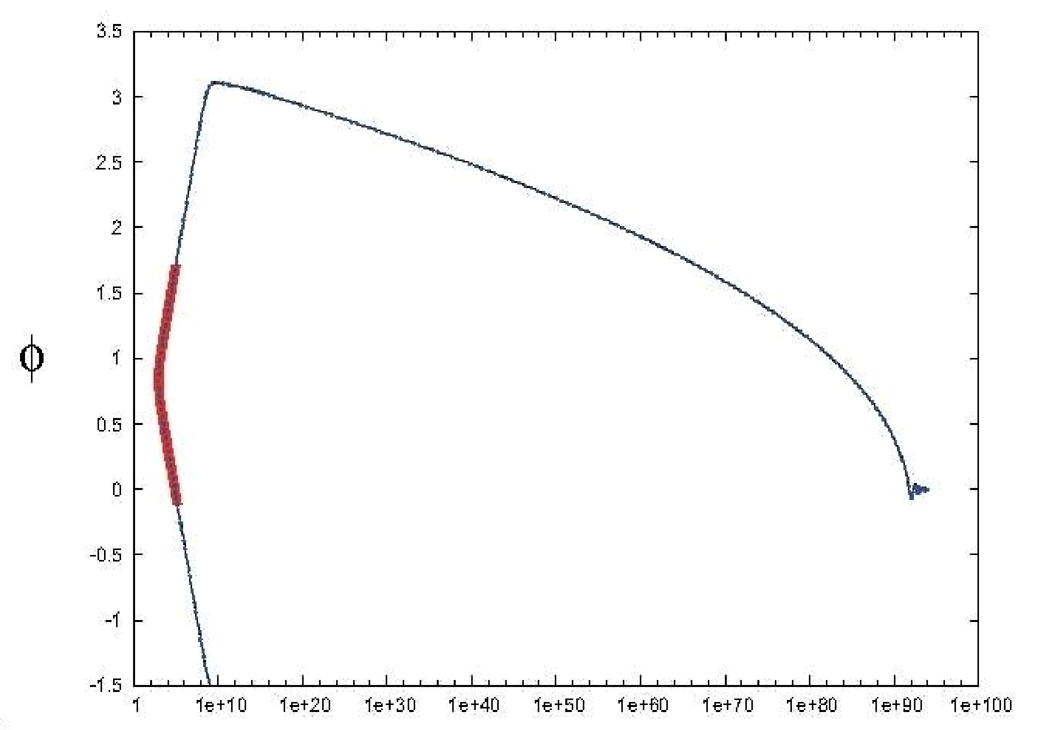
- The Hubble parameter is bounded from above.

- When the potential is bounded from below (e.g. $m^2\phi^2$), the time derivatives of the inflaton and the Hubble rate are bounded from above in norm ($|\dot{\phi}|, |\dot{H}|$).

- For potentials of the field unboundedly large at infinity, there exists an upper bound on the value of the inflaton.

- There exists a phase of **superinflation** (Bojowald, Singh) after the bounce.

- This superinflation does not yield sufficient **e-foldings**.



(Ashtekar)

LQC: Slow-roll inflation

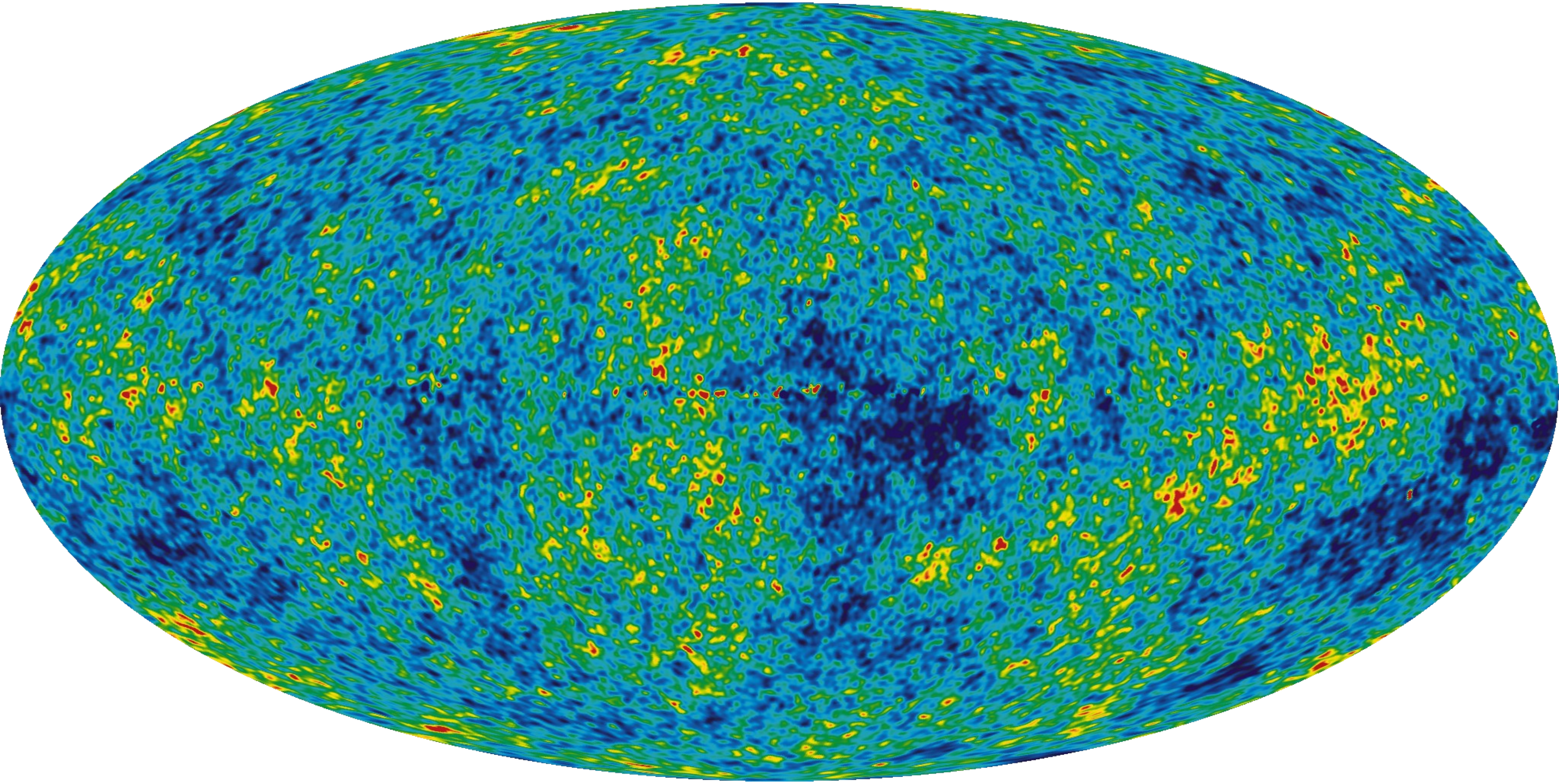
- Let us study the inflationary potential $m^2\phi^2$, with a mass of the order of 10^{-7} in Planck units (Ashtekar & Sloan).
- For standard initial conditions, the quantum corrections to GR at the onset of inflation are of a relative order 10^{-11} or less.
- We can calculate the **probability** that there are more than **68 e-foldings.** Starting with equiprobability for the unconstrained initial data, we get on the constraint surface (and gauge section of the bounce) the Liouville measure:

$$d\mu_L \propto \sqrt{1-F_{Bounce}}\, d\phi_{Bounce}\, dv_{Bounce}.$$

- $|\phi_{bounce}|$ is **bounded**, because the potential grows unboundedly.
- F_{Bounce} is the fraction of the matter density in the potential at the bounce.

- If $F_{Bounce} > 1.4\ 10^{-11}$ the superinflation phase provides the desired number of e-foldings or initial conditions to have a sufficiently long slow-roll inflation.
- For this range of F_{Bounce}, the relative probability is greater than **0.99.**

LQC: Inhomogeneities?



LQC: Inhomogenous models

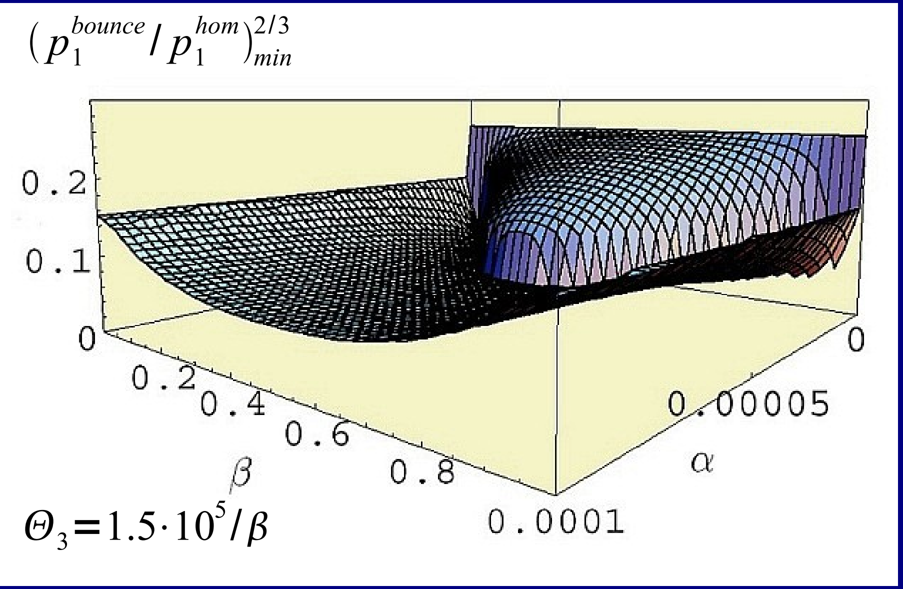
- **HYBRID APPROACH** (Garay, Martín-Benito, Mena Marugán, Martín-De Blas):
  - Loop quantization of the homogenous degrees of freedom of the geometry.
  - Fock quantization of matter fields and inhomogeneous gravitational waves.
- One assumes a hierarchy in the relevance of quantum geometry phenomena.
- There are recent results about the choice of a UNIQUE Fock quantization in cosmological scenarios (Cortez, Mena Marugán, Olmedo, Serôdio & Velhinho...).
- Systems in this category are GOWDY cosmologies (with different topologies and possibly scalar fields), and fields and perturbations around closed FRW.

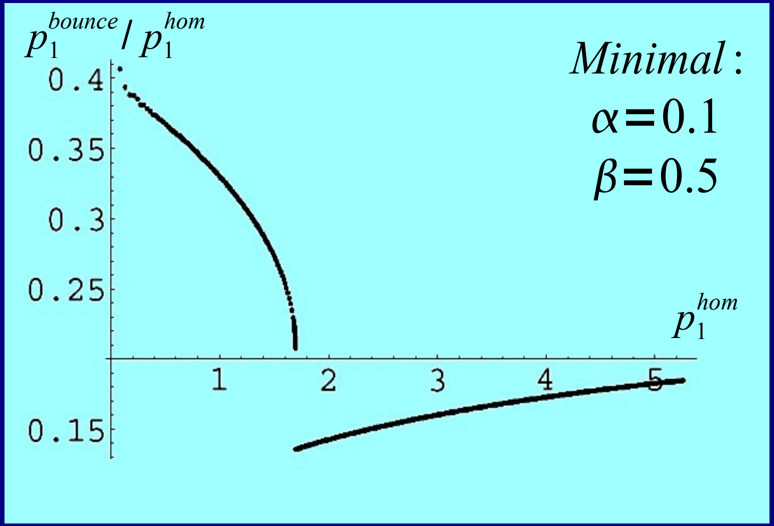
- One reaches a kinematical **RESOLUTION** of the cosmological singularity.
- The quantization has been **COMPLETED.**
- On physical states one **RECOVERS** the Fock space for inhomogeneities.

Hybrid Gowdy: Effective dynamics and Big Bounce

- For hybrid Gowdy (with T^3 topology) we have studied the effective dynamics. The homogenous background is a Bianchi I Universe. The prescription for improved dynamics has scaling problems, though. (Brizuela, Mena Marugán & Pawlowski).

The numerical analysis confirms the bounce in all three directions. The bounce happens typically at p's which are at least 13% those found without inhomogeneities.





$$\alpha = \frac{H^{\xi}_{Inter}}{H^{\xi}_{0}}, \quad \beta = \frac{\Theta_3}{\Theta_2 + \Theta_3}, \quad 0 \le \frac{\alpha}{2}, \beta \le 1.$$

$\Theta_I = C_I p_I$. Θ_2 and Θ_3 are constants of motion.

Gowdy: Big Bounce and inhomogeneities

- We have studied the **change** in the amplitudes of the inhomogeneous modes (describing linearly polarized gravitational waves) between the two asymptotic regions corresponding to contracting and expanding universes.

- We have taken an statistical average, disregarding phases.

- In the sector where the **inhomogeneities dominate** the bounce dynamics, the change in the amplitudes is antisymmetric with respect to their phase.

  Then, the amplitudes are statistically **preserved** through the bounce.

- In the sector where the **vacuum dynamics** is approximately valid around the bounce, the change is **positive** in average.

  Hence, the bounce **pumps energy** into the inhomogeneities.

- Although the scenario is not completely physical, the behavior may indicate a LQC mechanism to remove low amplitudes.

Concluding remarks

- Other approaches to deal with inhomogeneities have been developed by Rovelli & Vidoto and by Bojowald et al.

- LQC allows one a rigorous control on the mathematical and interpretational aspects of cosmology.

- LQC has opened new views to the quantum phenomena of the Early Universe:

  - Resolves the Big Bang singularity
  - Leads to a Big Bounce that respects "semiclassicality".
  - Renders inflation a natural process.
  - Provides a new setting to discuss initial conditions for the inhomogeneities.
  - Might remove low amplitudes from the inhomogeneities.

- Further research should deal with inflation, inhomogeneities and perturbations.

- Many others have contributed to LQC (Lewandowski, Kaminski, Banerjee, Date, Khanna, Hossain, Shankaranarayaran, Szulc, Varadarajan, Yongge Ma...)