Correlation Functions in Black Holes and White Holes Formed by Bose-Einstein Condensates

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Index

Index

1. Motivation
2. Black Holes and White Holes
3. BECs
4. Correlations in Black Holes and White Holes

Motivation

- Black holes emit thermal radiation (Hawking)

$$T_H = \frac{\hbar c^3}{8\pi M G k}$$

- For $M = M_\odot$, $T_H \sim 10^{-6} K << T_{CMB} \sim 3K$

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  - Hawking quanta $(E > 0)$ in the exterior
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- Pair creation in the near horizon region:
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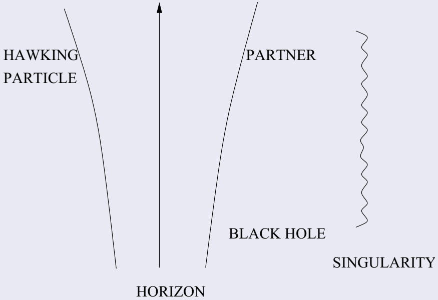
  - Partner $(E < 0)$ in the interior
- Analog models (Unruh):
  Sound waves in a supersonic fluid behave as a scalar field in a black hole geometry (acoustic metric)
- An analog thermal emission of phonons is expected
- Both acoustic black holes and white holes configurations are physically realizable (we will consider Bose-Einstein condensates)
- We shall study correlations between the Hawking quanta and partner

Index

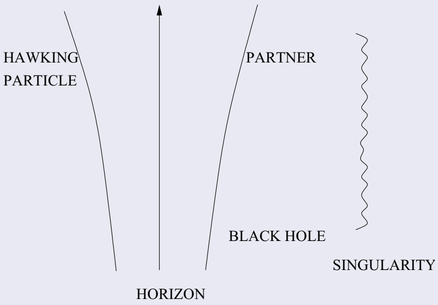
Black Holes and White Holes

Black Hole

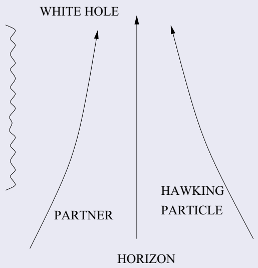


Black Holes and White Holes

Black Hole



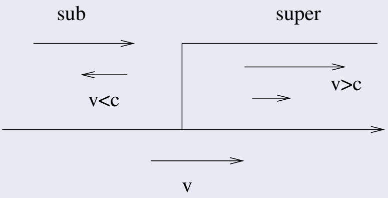
White Hole



- Time reversal of BH

Analogue Configurations

Sonic Black Hole



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- Negative energy states exist in the sup region

Analogue Configurations

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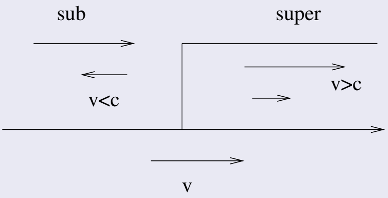

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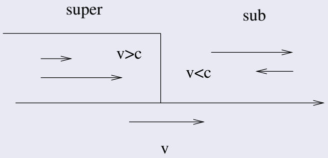
Sonic White Hole



- Sound waves cannot remain in the supersonic part
- Again negative energy states exist in the sup region

Index

BECs. Basic equations

- A Bose Gas can be described by $\hat{\psi}$,

$$[\hat{\psi}(t,\vec{x}), \hat{\psi}^\dagger(t,\vec{x}')] = \delta^3(\vec{x}-\vec{x}')$$

and

$$i\hbar\partial_t\hat{\psi} = \big(-\frac{\hbar^2}{2m}\vec{\nabla}^2 + V_{ext} + g\psi^\dagger\hat{\psi}\big)\hat{\psi}$$

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$$i\hbar\partial_t\hat{\psi}=\big(-\frac{\hbar^2}{2m}\vec{\nabla}^2+V_{ext}+g\psi^\dagger\hat{\psi}\big)\hat{\psi}$$

If $\hat{\psi}\sim\psi_0(1+\delta\hat{\psi})$ (ψ_0 is the condensate wavefunction and $\delta\hat{\psi}$ are the fluctuations)

ψ_0 satisfies the Gross-Pitaevski equation

$$i\hbar\partial_t\psi_0=\big(-\frac{\hbar^2}{2m}\vec{\nabla}^2+V_{ext}+gn\big)\psi_0\ ,$$

while the fluctuation satisfies the Bogoliubov-de Gennes equation

$$i\hbar\partial_t\hat{\delta}\hat{\psi}=\big(-\frac{\hbar^2}{2m}\vec{\nabla}^2-\frac{\hbar^2}{m}\frac{\vec{\nabla}\psi_0}{\psi_0}\vec{\nabla}\big)\delta\hat{\psi}+mc^2(\delta\hat{\psi}+\delta\hat{\psi}^\dagger)$$

Density-density correlation function

The basic quantity to compute is

$$G^{(2)}(t;x,x^{'}) = \frac{<: n(t,x)n(t^{'},x^{'}) :>}{< n(t,x) >< n(t^{'},x^{'}) >}\Big|_{t=t^{'}},$$

where

$$n(t,x) = \psi^{\dagger}(t,x)\psi(t,x)$$

BECs non linear (superluminal) dispersion relation

Dispersion effects for the fluctuations

$$(\omega - vk)^2 = c^2k^2 + c^2k^4\xi^2/4$$

are important when $k >> 1/\xi$ ($\xi = \hbar/(mc)$ is the healing length)

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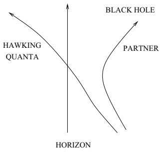
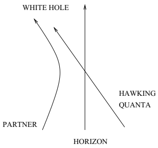
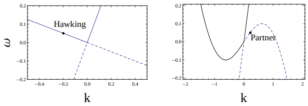
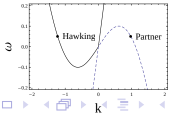
are important when $k >> 1/\xi$ ($\xi = \hbar/(mc)$ is the healing length)
The trajectories of the Hawking quanta and partner are modified





The main contribution to the correlations comes from the small ω region, which for the white hole is far from hydrodynamics ($k \simeq k_0 \neq 0$)

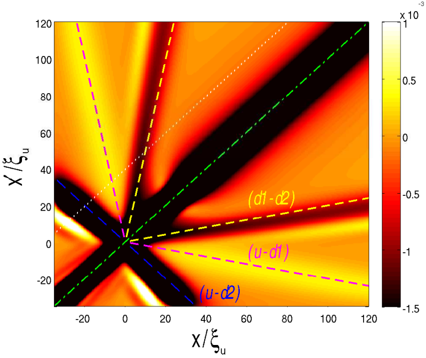




Index

Correlations

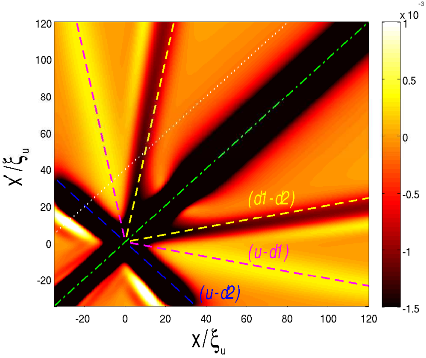
Black hole



Hawking-partner correlation is
$u - d2$

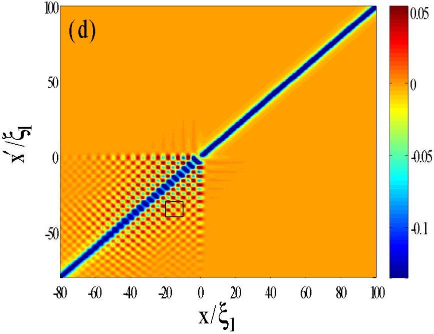
Correlations

Black hole



Hawking-partner correlation is $u - d2$

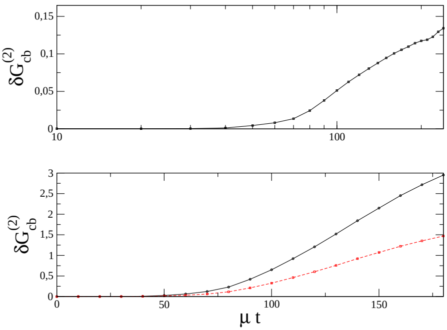
White hole



Hawking-partner correlations give a checkerboard

Correlations. Dependence in time.

Whereas in the BH case the signal is stationary, surprisingly in the WH we have a correlation growing in time



- $T = 0 \Rightarrow$ Growth in $\ln(t)$
- $T \neq 0 \Rightarrow$ Growth $\propto t$

Final comments

- Differently from the BH, in the WH case as time goes on correlations seem to grow without bound.
- Also the mean density seems to grow with time.
- This can be interpreted as a (mild) instability...

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Thanks