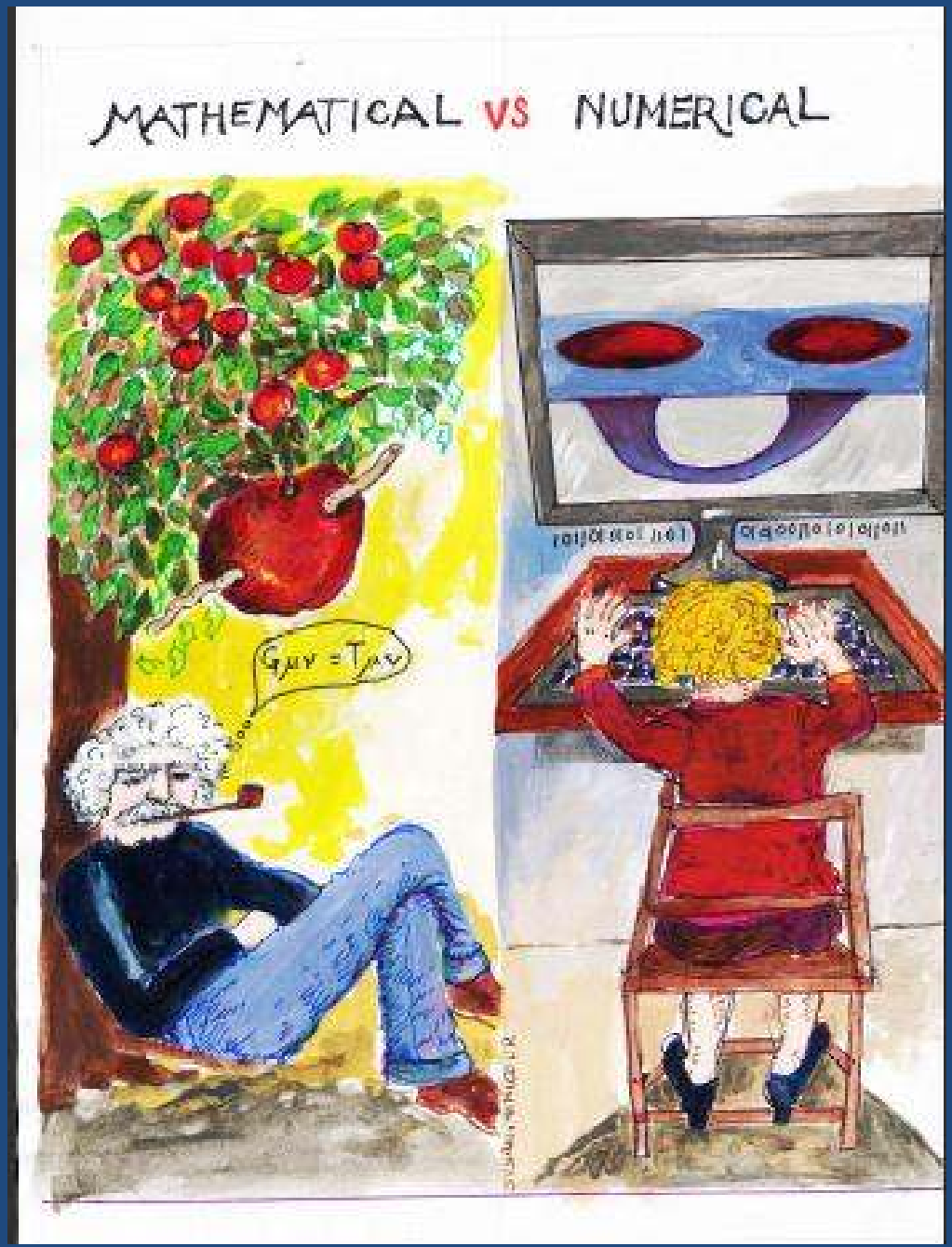

MATHEMATICAL VS NUMERICAL

Numerical Relativity as a tool for discovery

L. Lehner

(a.k.a 'the replacement')

(Perimeter Institute/Univ. of Guelph/CIFAR)

NR...why...*If we think hard enough we won't need a computer*

With the right resources we can simulate situations we can't even begin to think through, and thereby provide us with ***completely new and unexpected*** *things to think about*

- Support?
  - **Critical Phenomena in GR**;
  - Toroidal E.H.;
  - BH stability
  - Binary black holes, approach to singularities
- Goal: *Numerical construction of spacetimes.*
  - Access strong field/highly dynamical scenarios
    - Singularity structure
    - Collapse, merger
    - Role in energetic phenomena
  - Global description
    - Asymptotic behavior, cosmology, 'holographic' correspondences

In cases.. NR ahead/on par/behind Math Rel

GW driven…

- Obviously, GW driven efforts are pretty much at the front,
  - GWs observations should come alive < 2020. 2010 AstroDecadal report (USA) couldn't have been more favorable. Even 1 LIGO might move down under….certainly lots to do:
  - BBH, while much has been learned, still some 'open' problems:
    - Higher mass ratios? Contact with other approaches. Can help in self-force problem? [Lousto-Zlochower,….]
    - Sweet spots for surprises?, 'super-radiance' regime?
    - Embed them in more 'natural settings', not for GWs… but for 'added physics'
    - Approximations?
  - BH-NS, NS-NS, in some sense in its infancy due to complex physics
    - Eqn of state? Magnetic fields?, cooling mechanisms? → they do affect GWs
    - Also, connect with longer term effects/behavior! With GR can't do it all… (multiple scale expansions?)

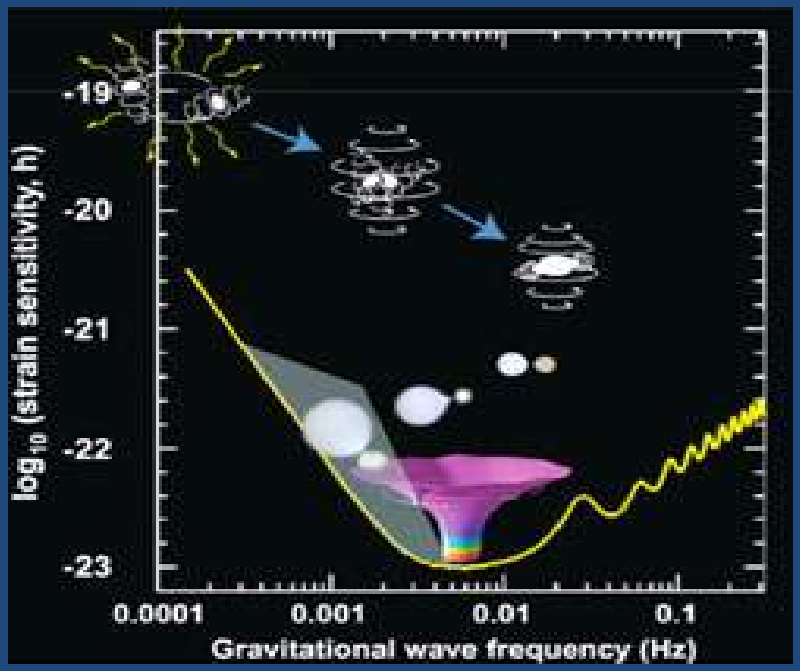
Beyond GWs

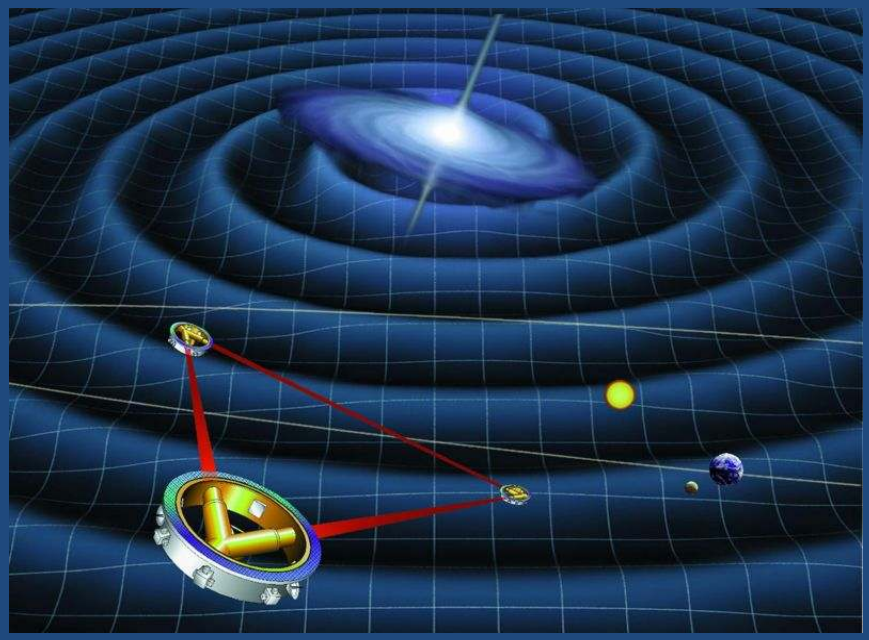
- A long list of fundamental qns still unanswered!
  - Nakedness? Ultra-spin/ultra-charge BHs? [Hubeny,Jacobson,Barausse.et.al 10]
  - Kerr BH stability [See Dain's talk]
  - Critical phenomena beyond spherical symmetry [Sorkin]
  - 'Micro-BH' formation and connections with higher dimensions [Sperhake et.al]
  - AdS/CFT and other dualities in truly dynamical scenarios
  - Cosmology: Dark matter/energy's role and observable consequences? Early universe behavior and imprints on GWs; alternatives to concordance model?, etc.
  - Strong gravity in higher dimensional settings?, new features/behavior beyond 4d intuition?

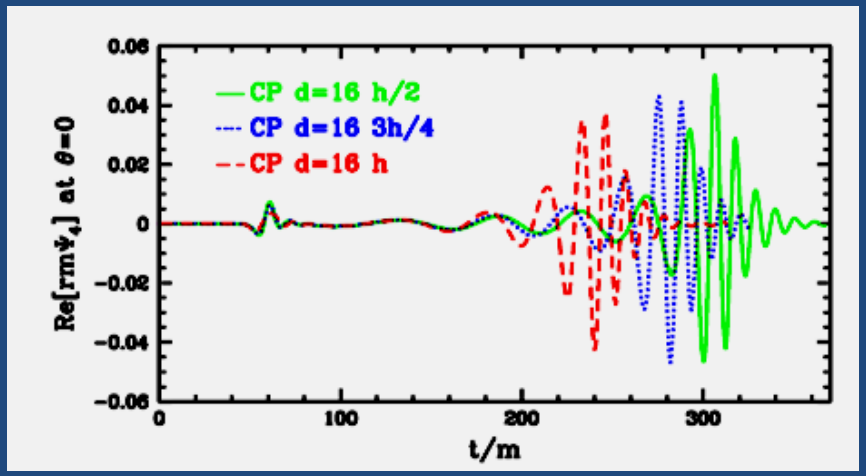
GWs & further physics

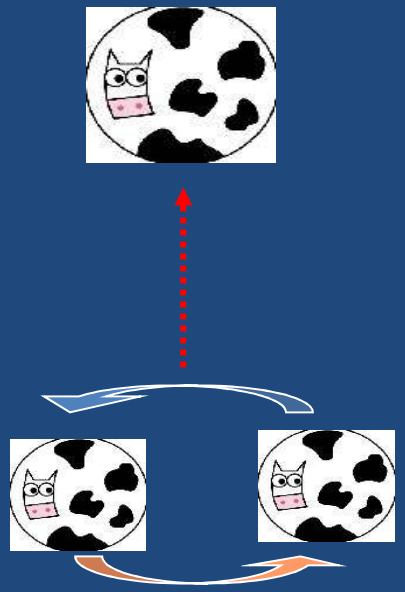
BBH and then some

Example I





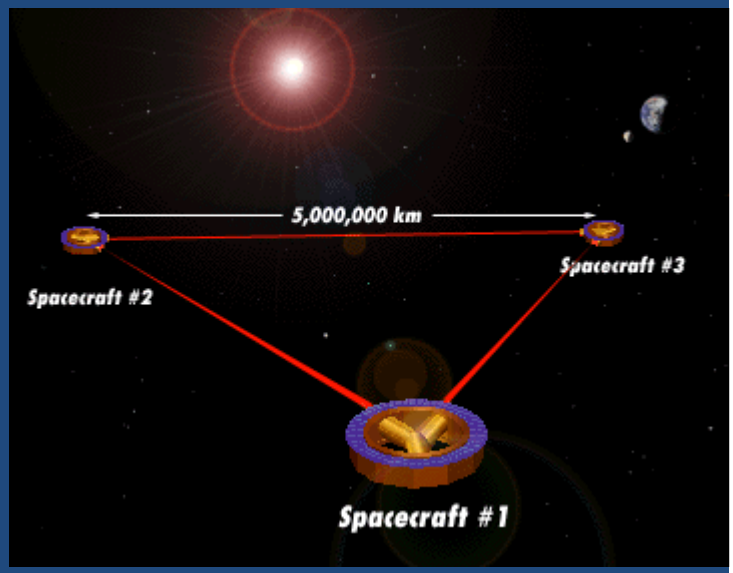




[Pretorius 05, …..]

Not so fast?

LISA: superb signal to noise ratio



- waves will be ``seen'' directly and to very large redshifts (z~ 5-10 …)

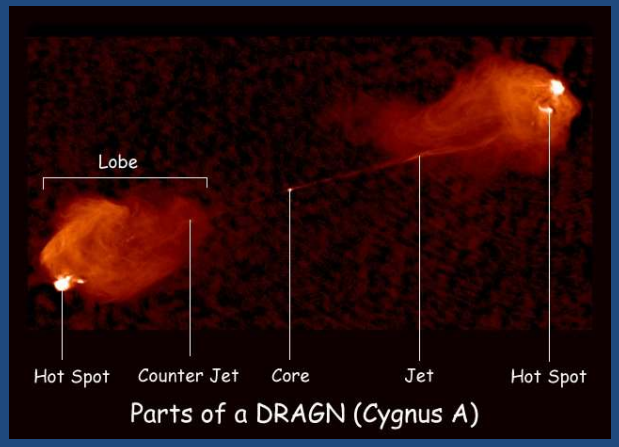
- Potential to do accurate astrophysics
- Potential to probe cosmology
- Potential to probe higher dimensional scenarios

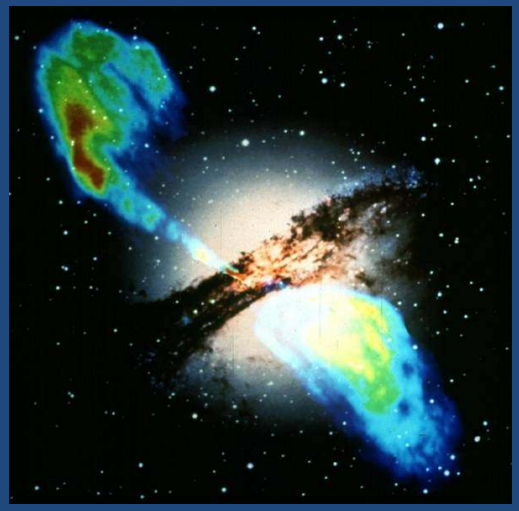
- However:
- localization to ~ square degrees [Holtz-Hughes]
- distance obtained is redshift dependent

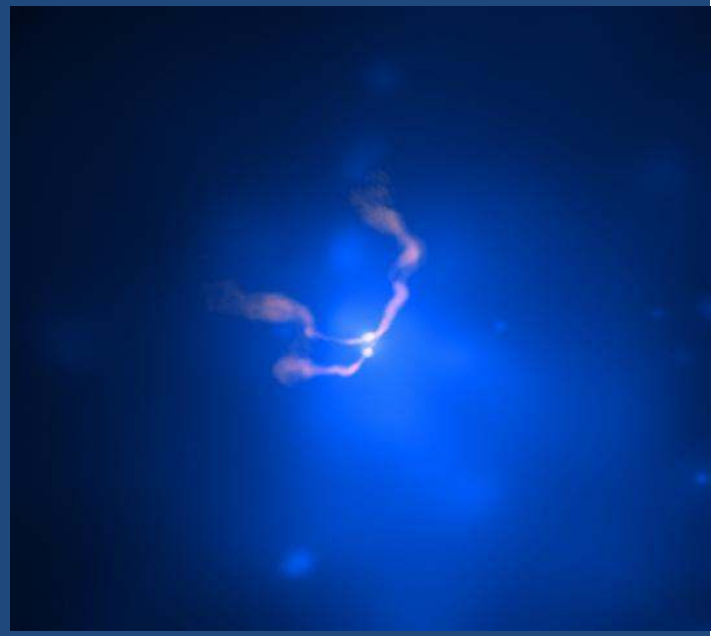
An electromagnetic counterpart resolves these issues

Nature cooperates…

- Understand both gravitational and electromagnetic wave emissions from key systems
  - Binary black holes interacting with surrounding media







10^{51} Ergs routinely inferred… even some 10^{61} ergs ?!

Studying relevant systems (BBH)

- Deal with spacetime curvature
  - Einstein equations. That's the 'solved' part! (ie… if you 'think' about it.. NR can likely give the answer, for comparable masses that is….)
- Black holes… are not really quite in vacuum…must deal with fields describing gas and electromagnetic fields
  - Poorly understood systems [we don't control the experiment]
  - Matter, what matter ?
  - Electromagnetic fields?
  - Emission process?

Two fronts.
(circumbinary picture)

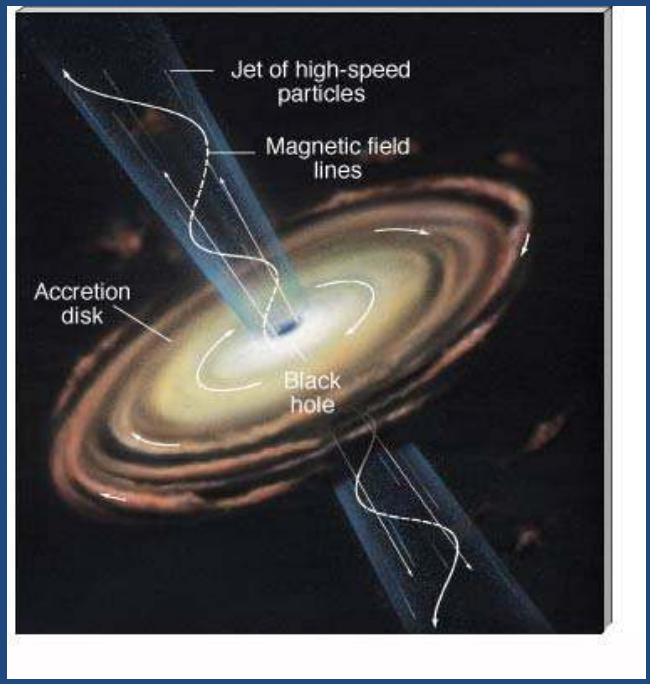
- Pre/prompt/post - merger emissions?
  - (pre/prompt) Binary black holes as EM field stirrers
  - (post) Binary black holes as bullies for matter

Merger of galaxies

-observations indicate the presence of supermassive BHs in the center of galaxies, surrounded by gas and an accretion disk

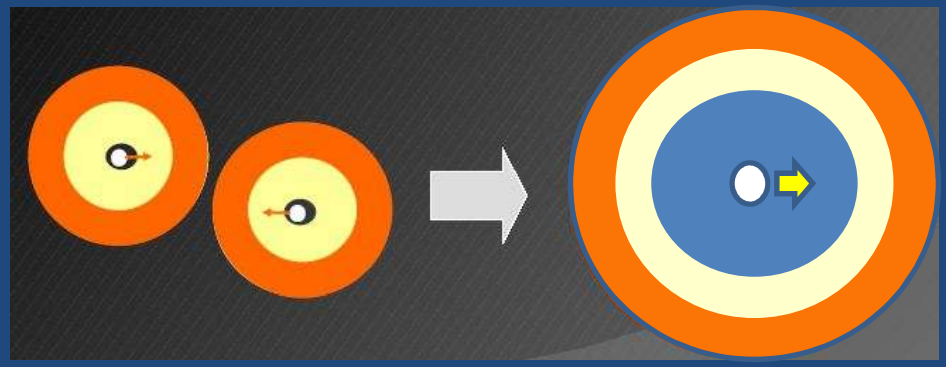
- these galaxies have undergone mergers → binary black hole merger

- further, AGNs → BHs are surrounded by a disc of matter likely magnetized.



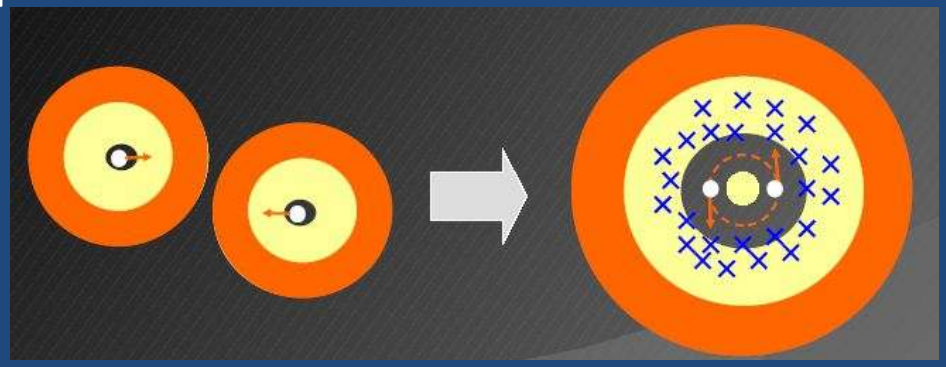
Binary black holes and emissions

- Different possible options.
  - Postmerger events from circumbinary disks around BHs



[Milosavljevic-Phinney; Lipai-Loeb; *Lipai et.al, Bonning et.al; Bode et.al; O'Neil et. al; Megevand et.al; Corrales et.al, etc.*]

  - Pre/merger events from gas/plasmas in between BHs / torques on disk



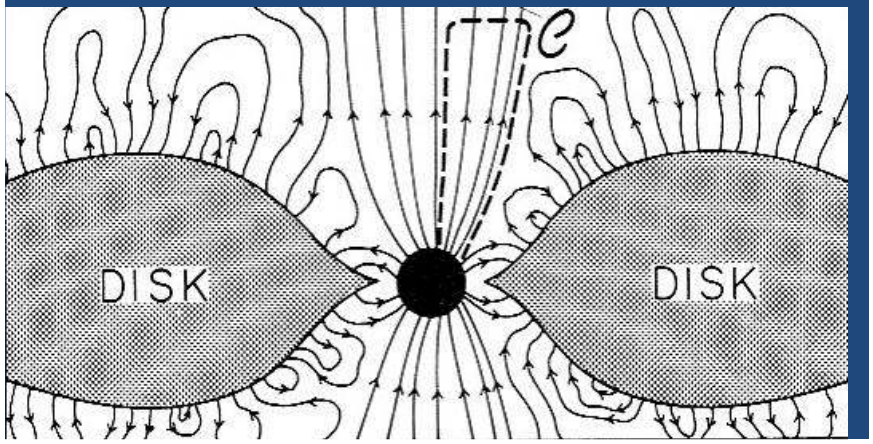
[Armitage et.al; MacFadyen et.al.; Dotti et.al; *Chang. et.al.; Palenzuela et.al.; Bode et.al…]*

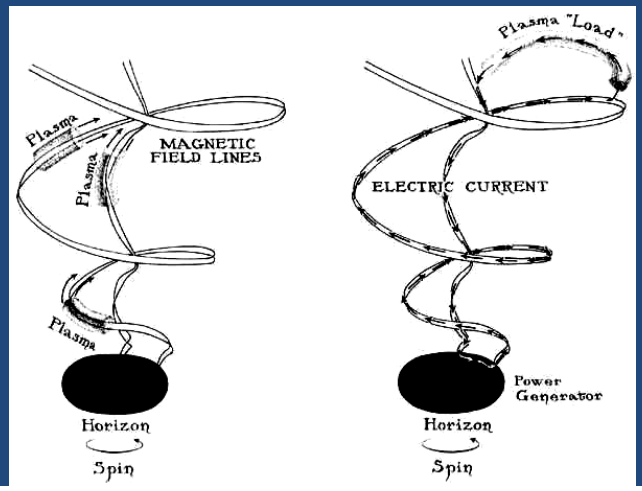
Binary black holes as blenders.
A new spin on an old story (though without spin)

How does the curvature/dynamics influence EM fields?

- Blandford-Znajek. “Penrose” process for Kerr bh’s surrounded by magnetic fields (anchored by the disk)

- Stray charges accelerate→ photons → pair production → cascade. BH becomes surrounded by a tenuous conducting plasma with little inertia





[Goldreich-Julian, Blandford-Znajek]

Approach: Force-free electrodynamics

$\nabla_a T^{ab}=0 \quad \rightarrow \quad \nabla_a T^{ab}{}_{(fluid)} = -\nabla_a T^{ab}{}_{(em)} = -F^{ab} J_a$

if $\rho, P \ll B^2$ then $\nabla_a T^{ab}{}_{(fluid)} \ll F^{ab} J_a \approx 0$

$\mathbf{E}\cdot\mathbf{J} = 0 \quad , \quad q\,\mathbf{E} + \mathbf{J} \times \mathbf{B} = 0 \rightarrow E\cdot B = 0$

Stationary spacetime:
(Gammie,McKinney 04)

$$E \bullet B = 0 \rightarrow F_{ab} * F^{ab} = 0 \rightarrow A_{\phi,\theta} A_{t,r} - A_{t,\phi} A_{\phi,r} = 0$$

so

$$\Omega_F \equiv -\frac{A_{t,r}}{A_{\phi,r}} = -\frac{A_{t,\theta}}{A_{\phi,\theta}} = \frac{F_{tr}}{F_{r\phi}} = \frac{F_{t\theta}}{F_{\theta\phi}}$$

$$E_{,t} = 2\pi \int_0^{\pi} \sqrt{-g} F_E d\theta \quad (with \quad F_E = -T_t^r)$$

$$\rightarrow F_E = 2(B^r)^2 \Omega_F r(a/2Mr - \Omega_F)\sin^2(\theta) - B^r B^\theta \Delta \sin^2(\theta)$$

thus,

$$F_E\big|_{r=r_H} = 2(B^r)^2 \Omega_F r_H (\Omega_H - \Omega_F)\sin^2(\theta)$$

$$\Rightarrow for \quad 0 < \Omega_F < \Omega_H \quad and \quad B^r \neq 0 \quad energy\ out\ of\ horizon$$

Plasma is crucial for this to happen

Examples…

- Kerr in vacuum and FF immersed in uniform field

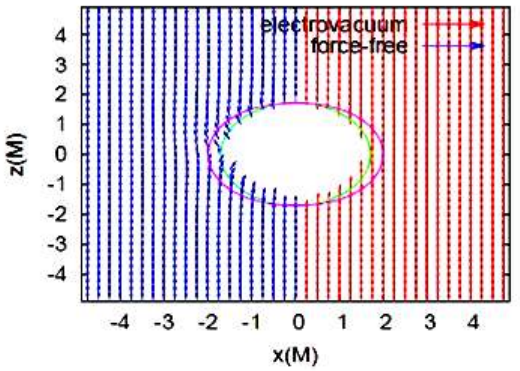
- In vacuum → no radiation

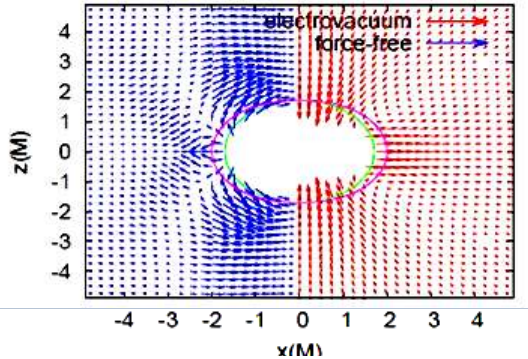
- With plasma → currents on the horizon ‘complete the circuit’

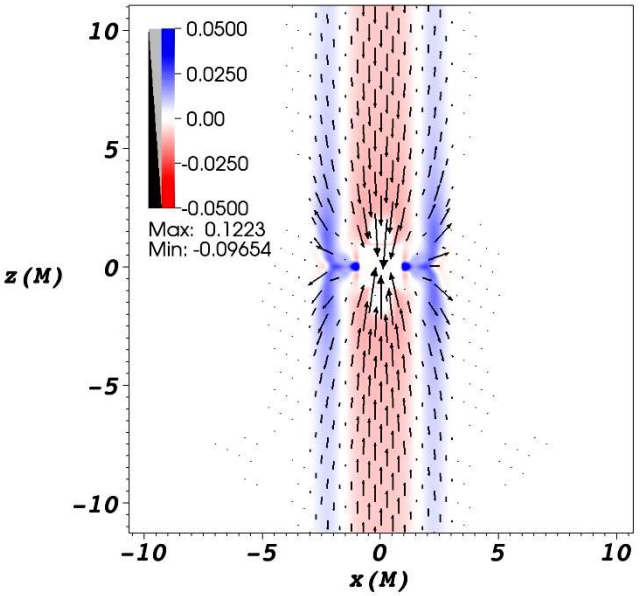
Membrane paradigm: wrt asymptotic observers, circuit moves through a B field → EMF produced.

BH becomes the battery.

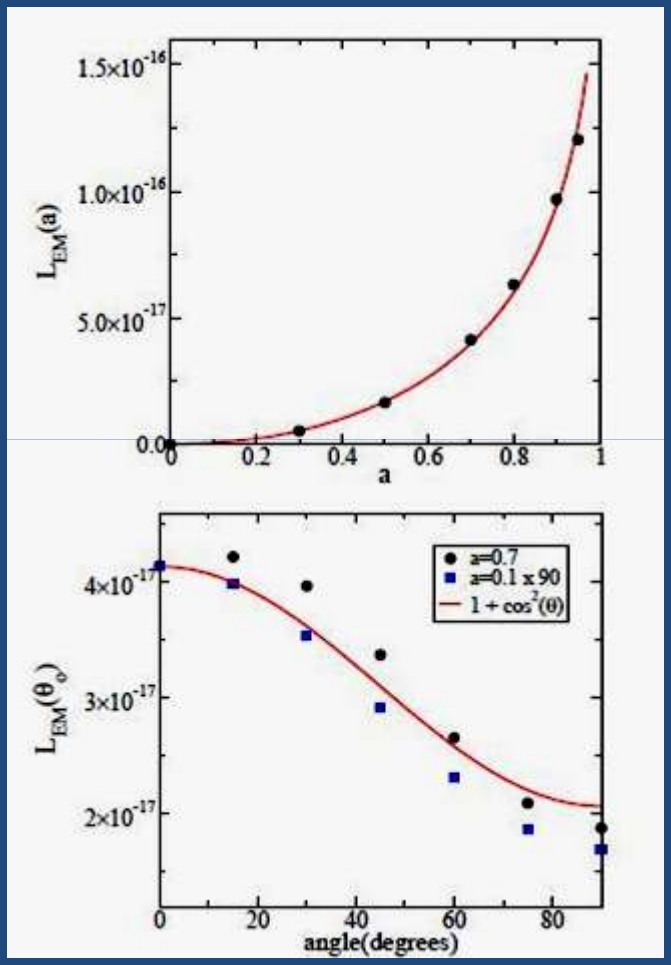
[Damour,Phinney,Thorne,McDonald…]

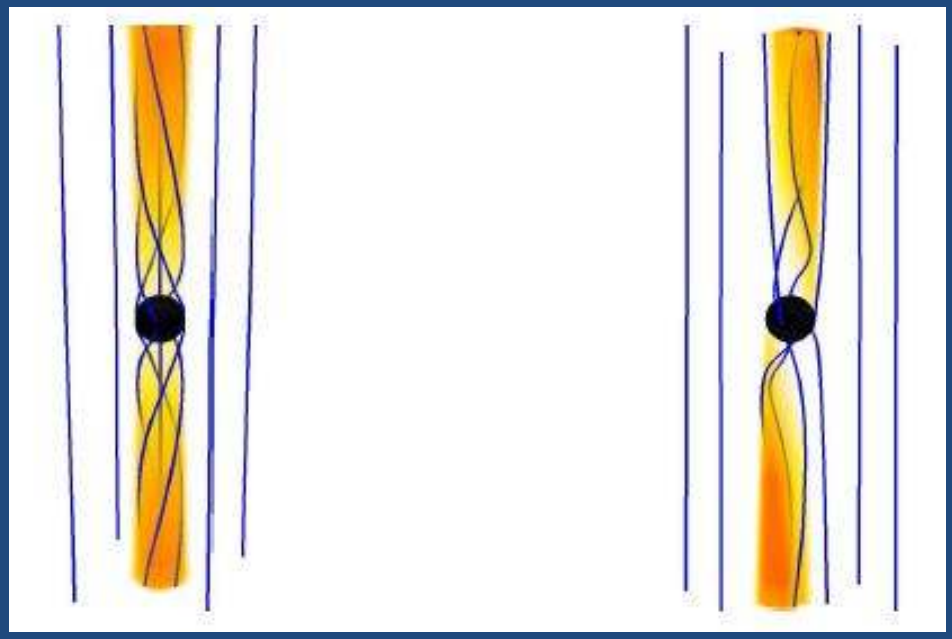






Single BHs, disk alignment?





- we knew. $P \sim B^2\ a^2$ in the aligned case [Tchechovskoy,Narayan,McKinney 2010].

- For misaligned case?
  - Poynting flux still there, along B
  - $P \sim B^2\ a^2\ (1 + \cos^2)$

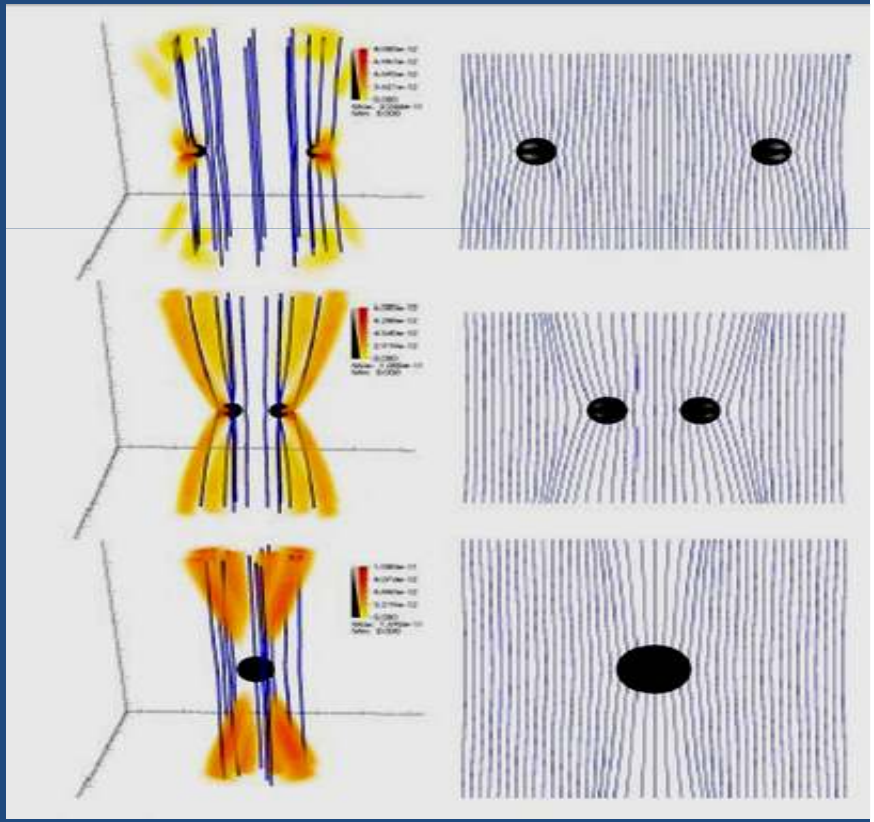
[Palenzuela,Garret,LL.Liebling, PRD 2010]

Onto binaries

- Head on & quasicircular, equal mass. *non-spinning*
- Magnetic field as given by a ‘circular loop’ at far distances ~ constant within computational domain
- Field strength ~ 10^4G
  - For this value, if $M_T=10^8 M_O$, EM Energy dens ~ 10^{-16} [1/M^2]
    - → EM fields won’t affect binary dynamics, but the other way around

Head-on case.

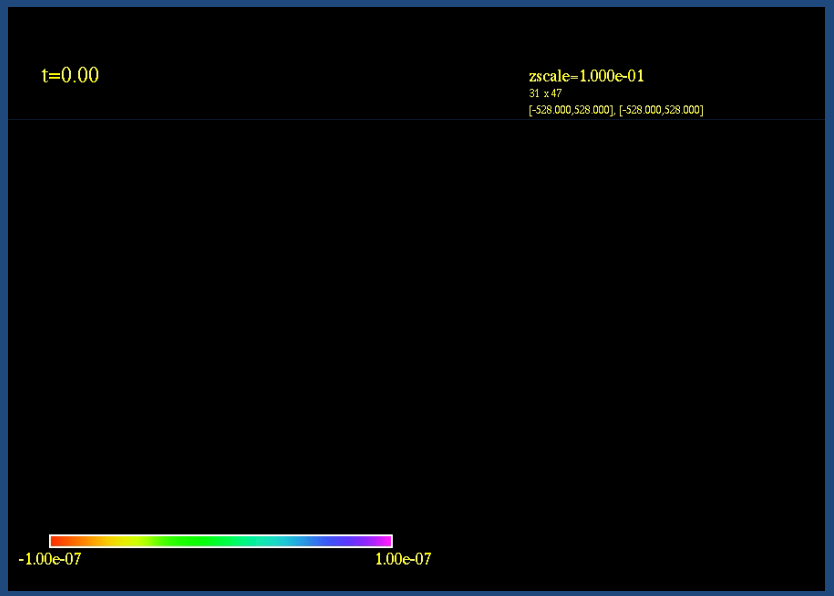
- Poynting flux,
  - What sources it ?
    - field lines tension/breaking as BH pulls them
    - Membrane paradigm: “Charge” separation induced by “Hall effect” , thus circuit is still there and still moving through B.
    - Poynting flux induced, though shuts off after merger

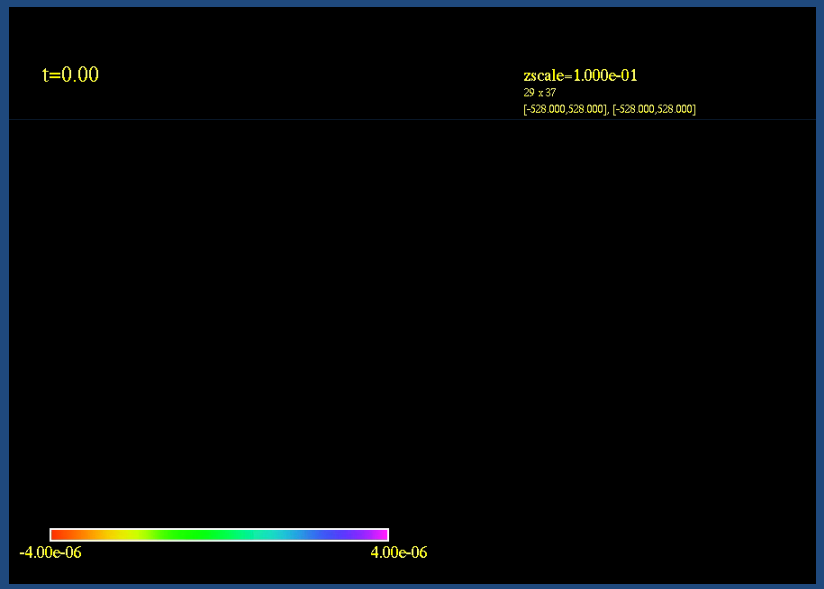


Onto the binary case

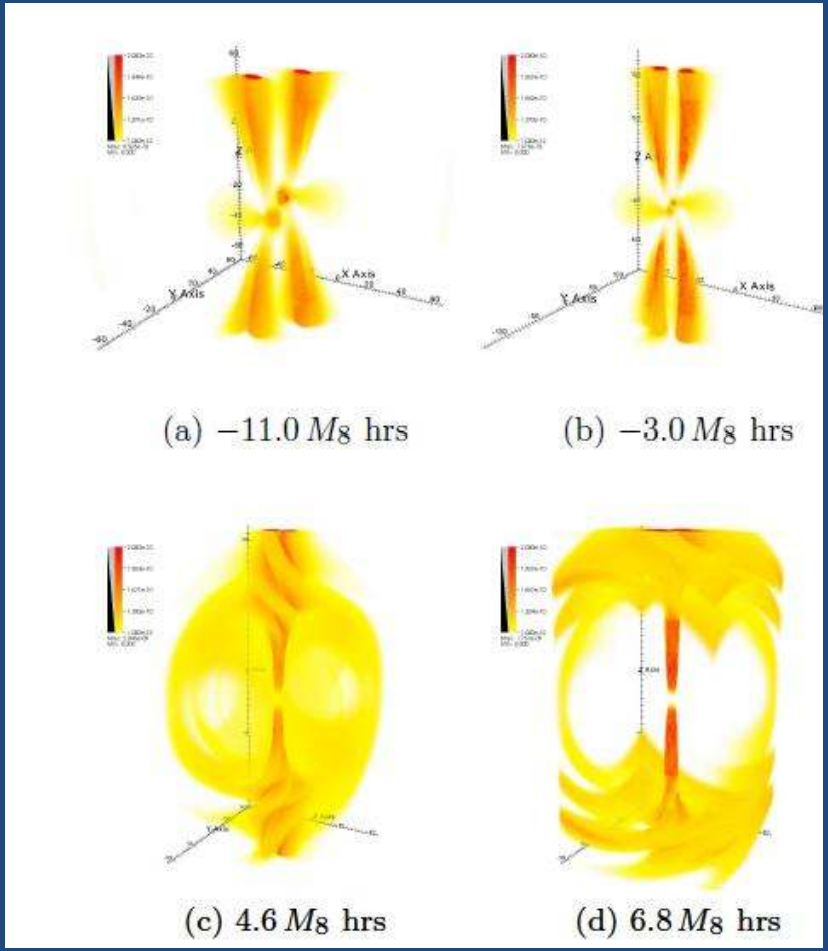
- Orbit → Black holes move through B. As in head-on case, 'circuit' can be Established due to charge separation (see in vacuum case already, [Palenzuela et.al. Moesta et.al.])

- Thus, expect Poynting flux through orbiting stages. Also at late time (BZ).

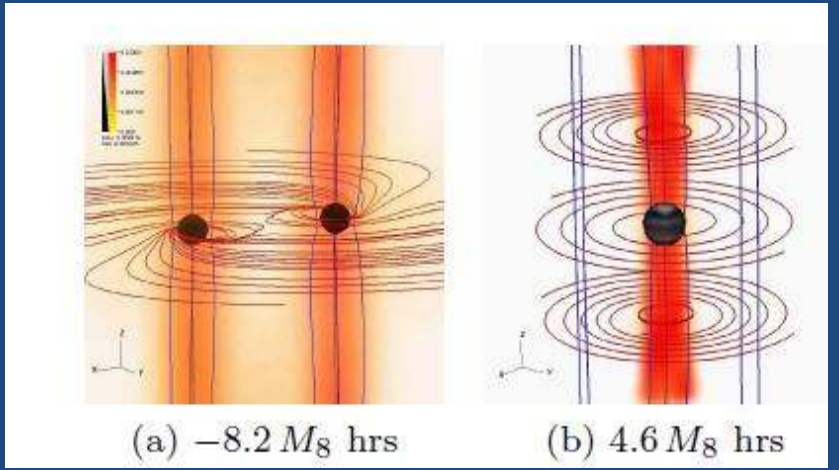




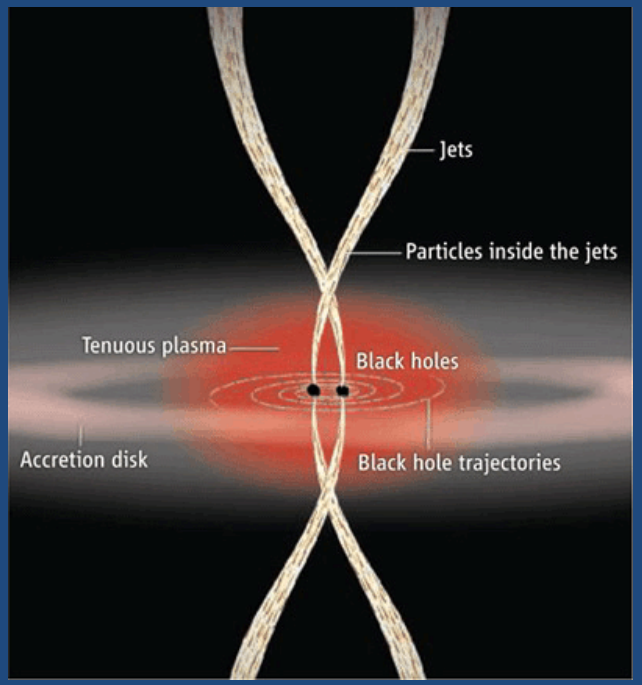
[Palenzuela,LL,Liebling , Science 2010]



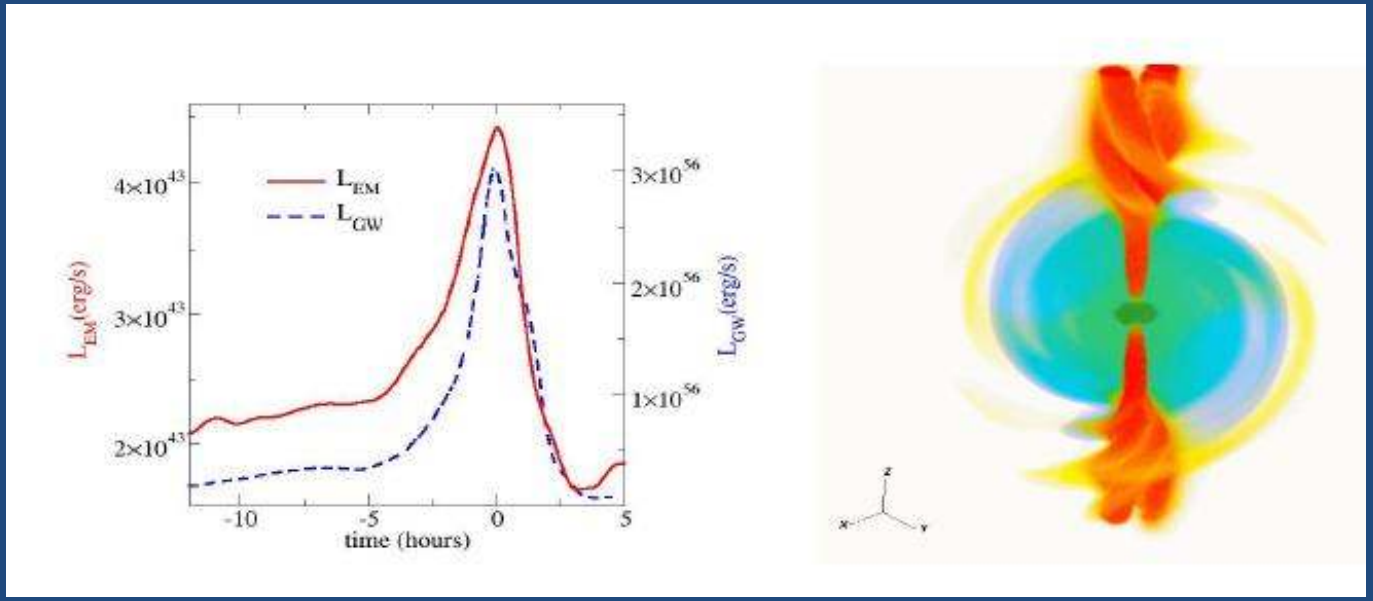
Poynting flux



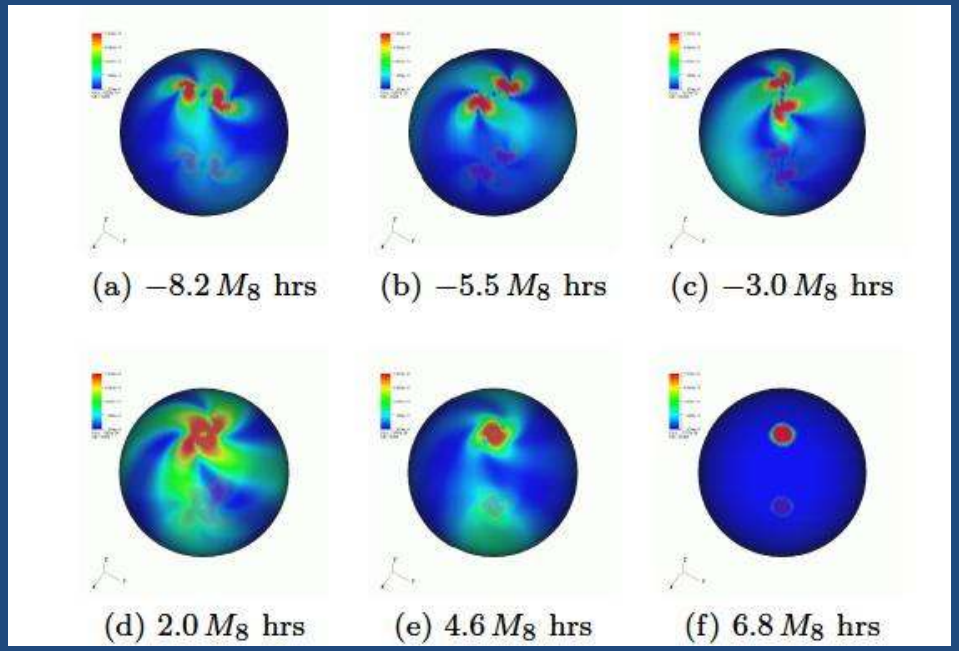
Ω_F



- Energy flux:



- Strong emission throughout. Burst around merger epoch



m=2 → 0 transition

Distributed energy output

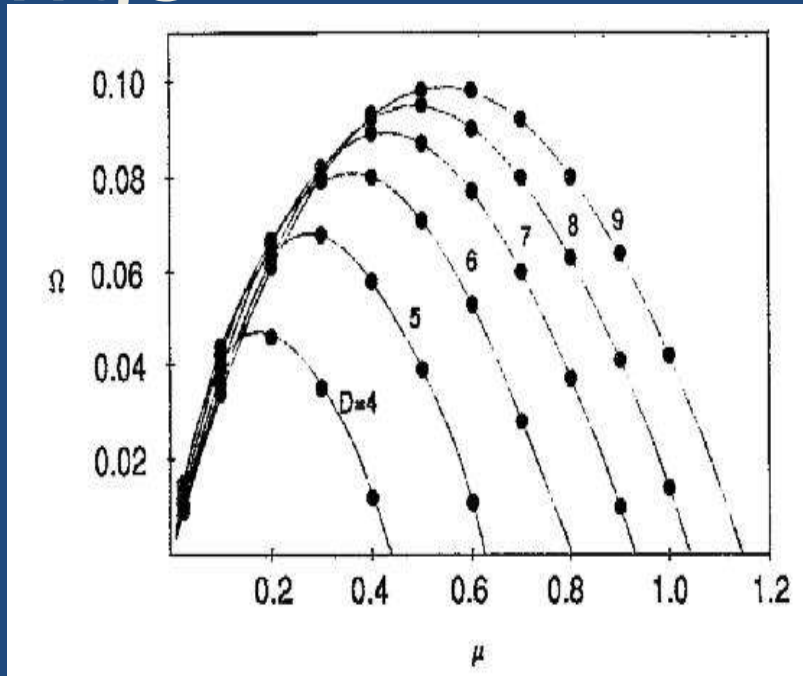
- Making contact with astro… recall $(R_{orb}\ \Omega_{orb}) < 1$
- GW energy flux ~ $R^4_{orb}\ \Omega^6_{orb}\ M^2$ --strong emission--
- EM energy flux ~ $(R_{orb}\ \Omega_{orb})^2\ B^2$ --weaker but sustained, doesn't shut off after merger--
- Spinning case will have BZ on top. Also, particularly 'cute' scenarios should show an interesting phenomenology
- For 10^4G, $10^8 M_O$ flux ~ $10^{43\text{-}44}$ ergs. IF Poynting flux energy efficiently transferred to observable emissions, interesting pre/post merger observations possible; to z=1 ?

BH stability, cosmic censorship

and interesting connections…

Black strings

1.- Contain singularities
2.- Ruled by null-rays
3.- Non-unique even in spherical symm

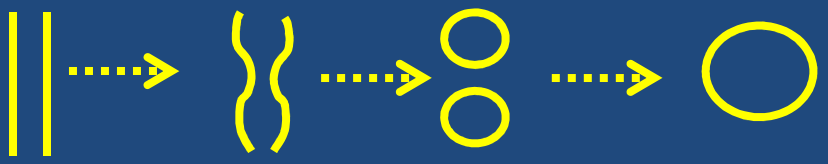


Stability?

- Black string perturbations admit exponential growth for $L > L_c$ (Gregory-Laflamme)

- Entropy $S_{BS} < S_{BH}$ (for a given M) [bs ~ M^2/L ; bh ~ $M^{3/2}$]

Conjecture: Black strings will bifurcate

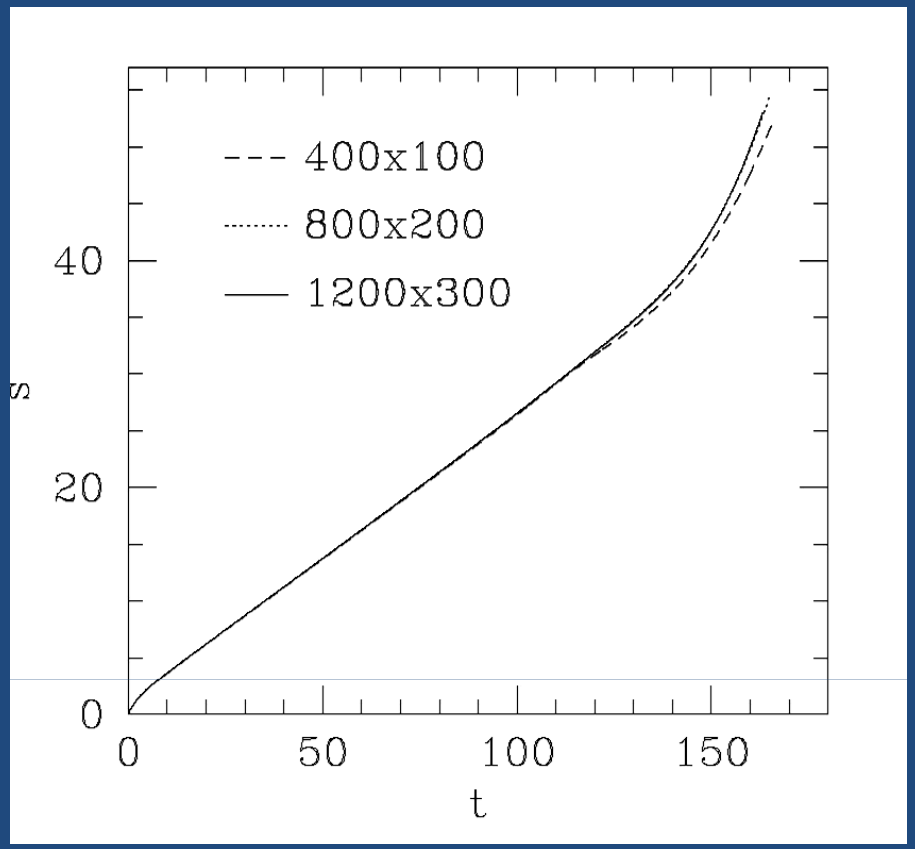


- Conjecture used in many scenarios
  - Density of states from Ads/CFT correspondence
  - Discussions of BH on brane worlds. BH in matrix theory, etc

Somewhat recent developments

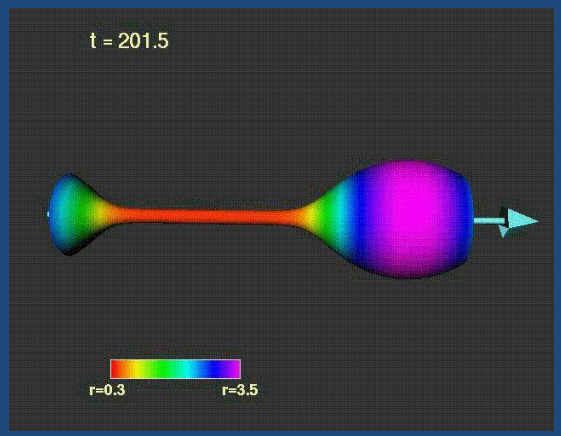
- Horowitz-Maeda, can't bifurcate in finite time. *Conjecture: will 'settle' to a non-uniform stationary soln*
- Gubser: transition to soln of first-order type in 5-6D (1^{st}, ~2^{nd} order pert)
- Wiseman: stationary solns which are not the Horowitz-Maeda ones (??)
- Kol: Transition from black string to BH through a conical singularity
- Sorkin-Kol: for high enough dimensions transition is of 2^{nd} order.

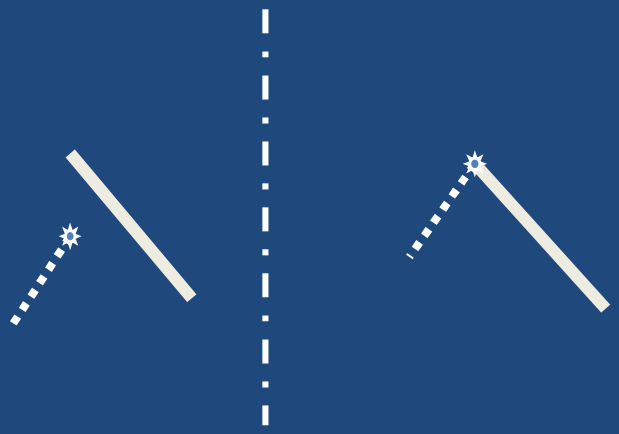
- Qns:
  - *What is the final solution of a perturbed black string?*
  - *Can it bifurcate in 'infinite time'?*
  - *Are Wiseman's solns, physically relevant?*



- Affine time, $\lambda = e^s$ growing exponentially ($\sim 10^{22}$)
- "bifurcation" in infinite affine time certainly possible
- 'cascade' of unstable strings also possible

[Garfinkle-LL-Pretorius]



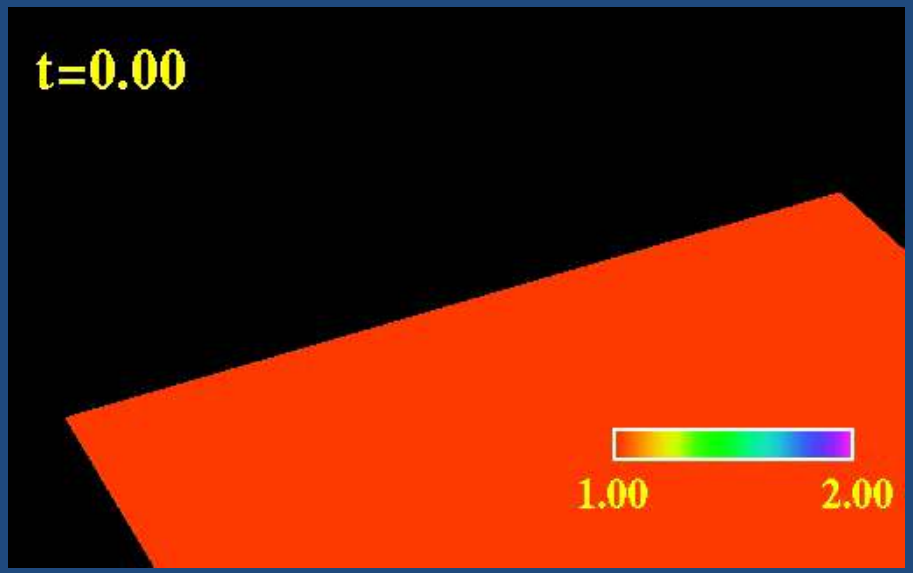


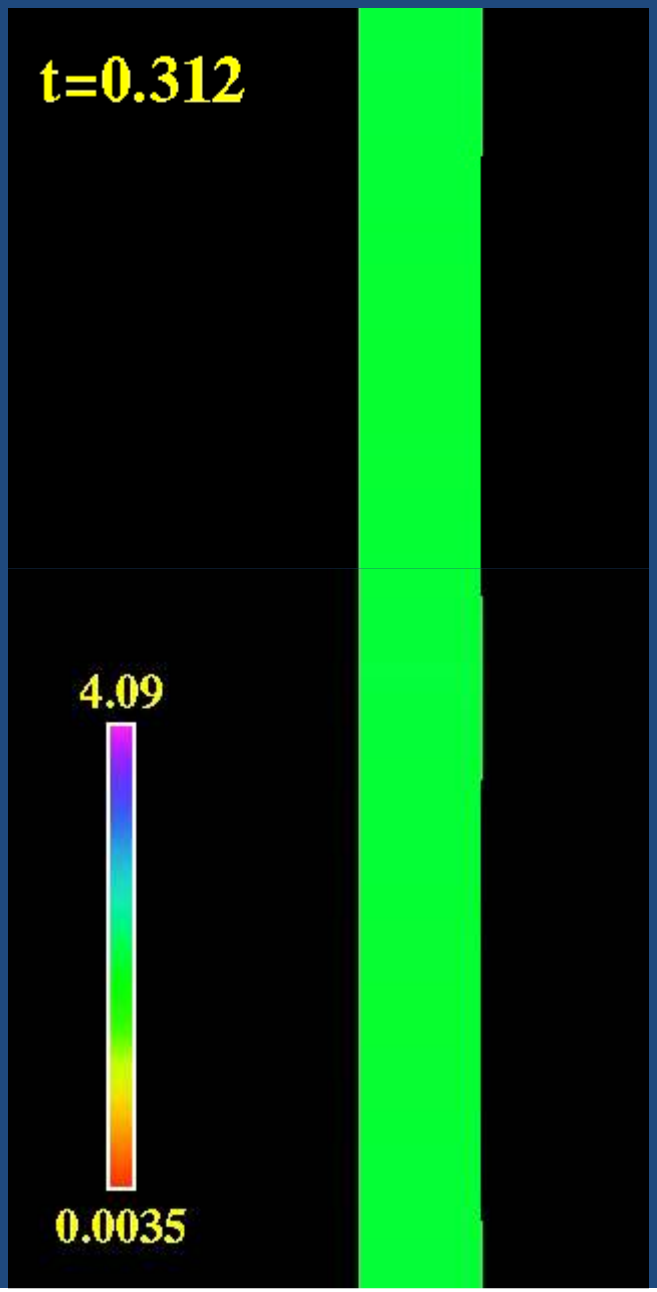
Finishing it up…[LL,Pretorius]

L/m = 20; L_c ~ 14 m

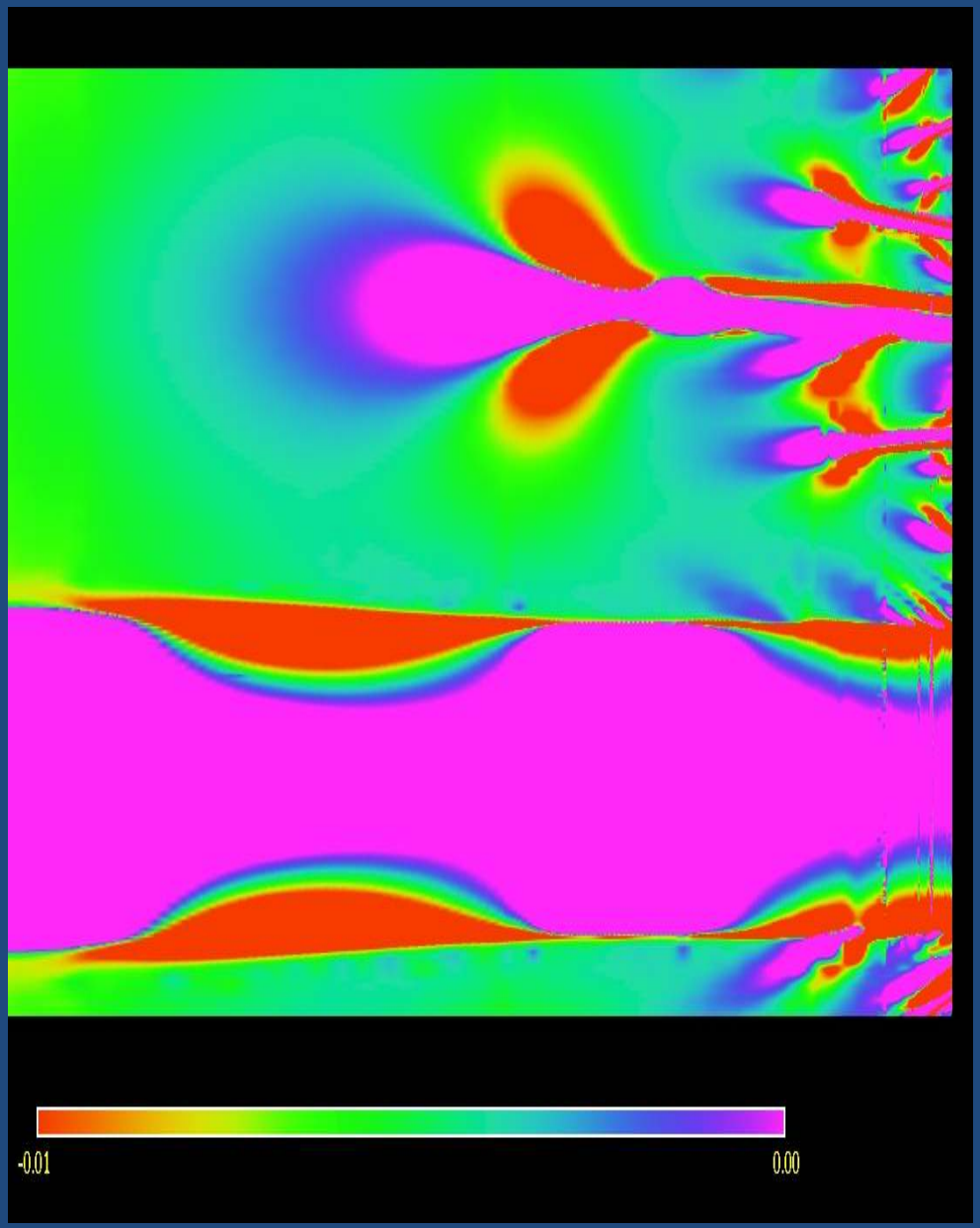
M = 4 π $(2m)^2$ L

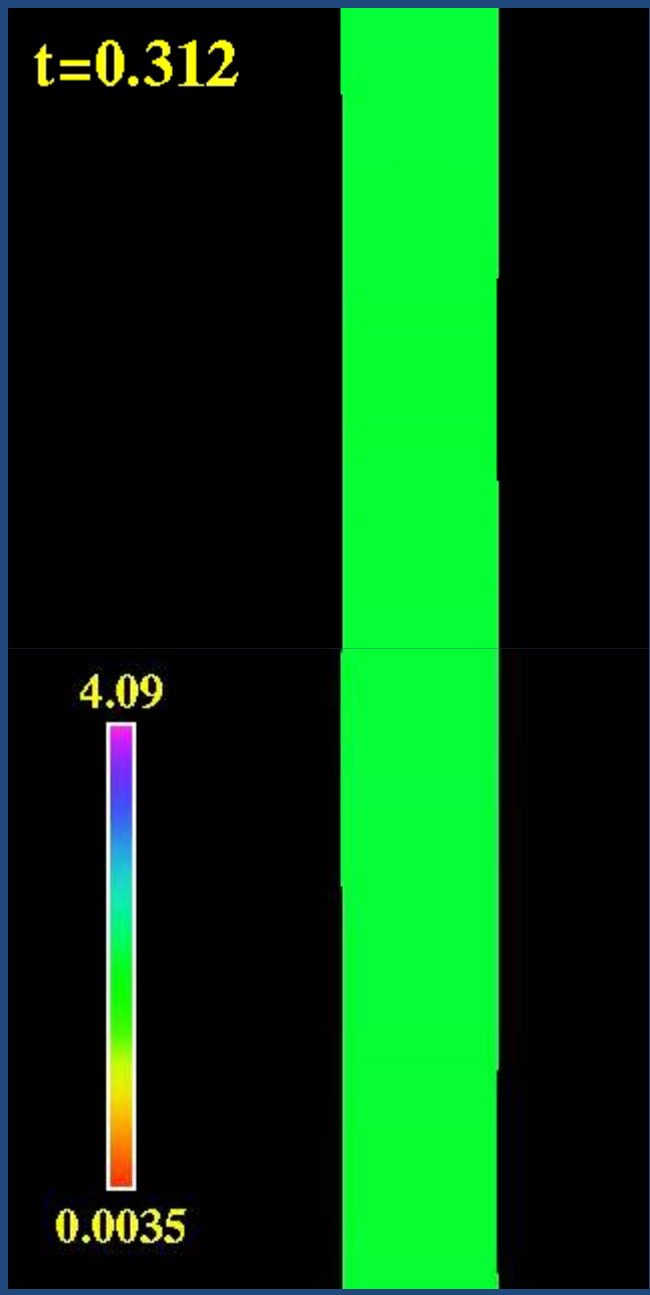
Btw…S_{BH}/S_{BS} = 1.374…



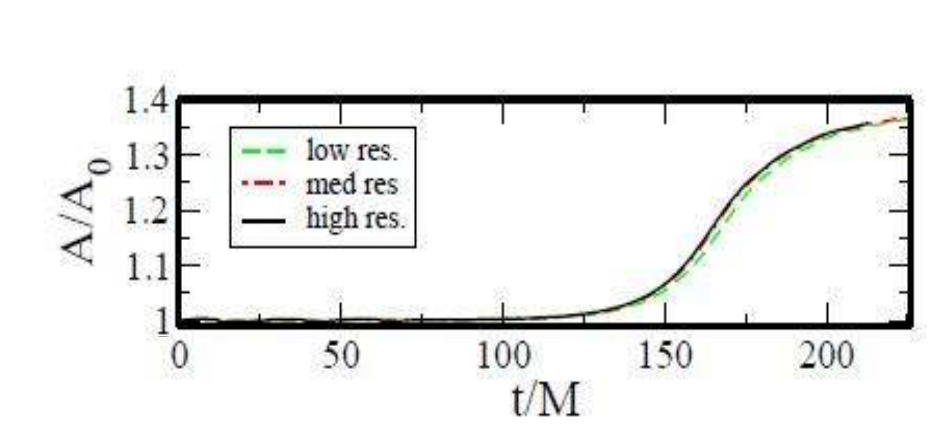


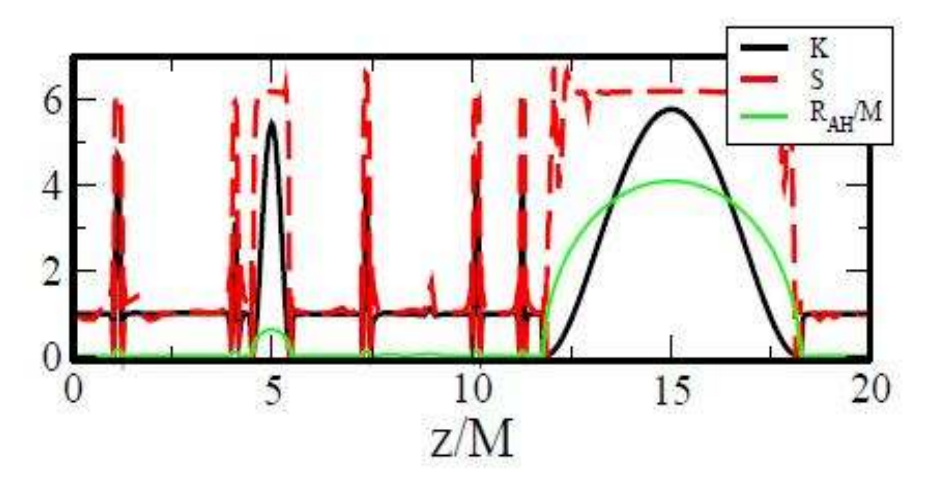
Zoom in..



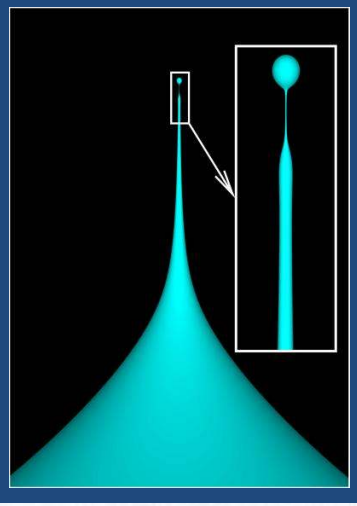


Details…





- S(late time) ~ 1.369 S(t=0)
- invariants.. bhs+ bss !
- Can calculate ‘thin-to-zero’ time T~ 231M
- → cross Planck length in finite time
- → local solution: M=0 BH (Choptuik critical phenomena). Without fine tuning → generic violation of cosmic censorship!
- Not to mention an analogy…



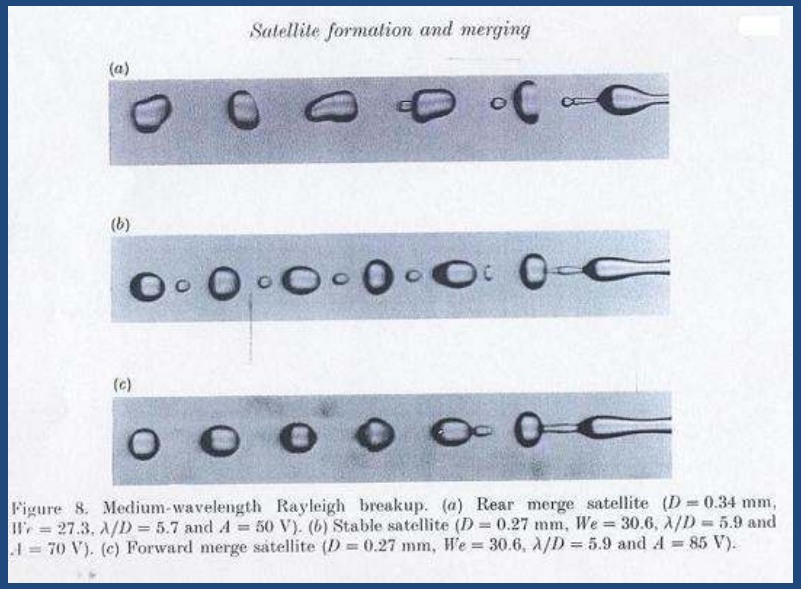
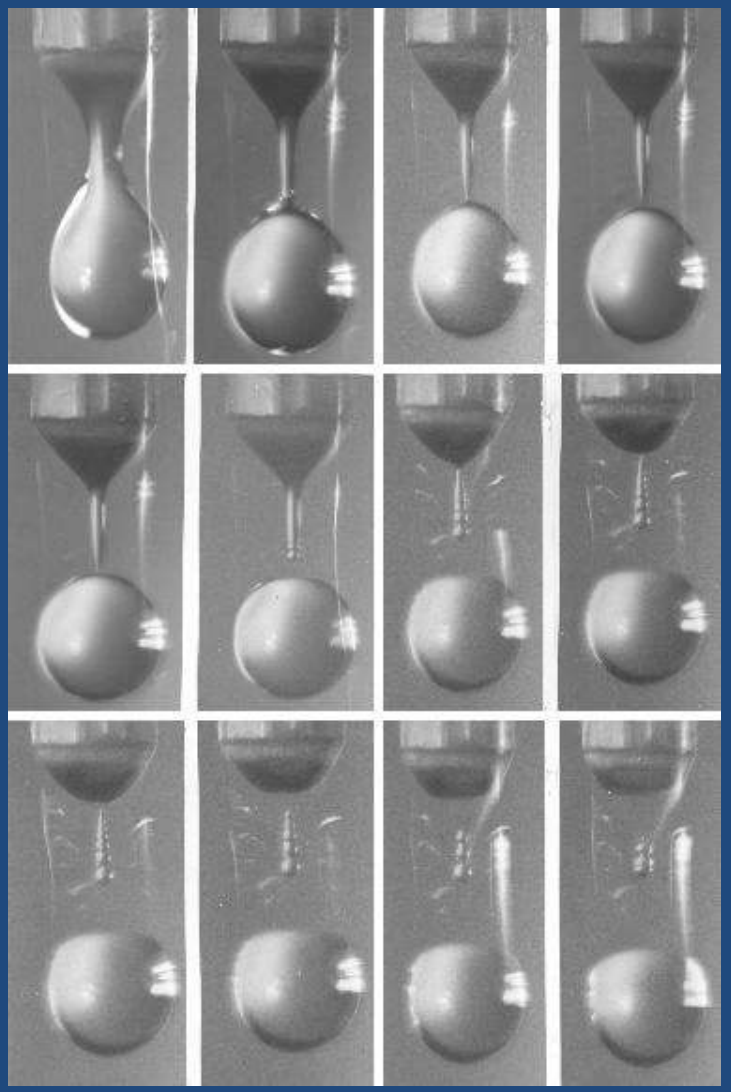


Figure 8. Medium-wavelength Rayleigh breakup. (*a*) Rear merge satellite ($D = 0.34$ mm, $We = 27.3$, $\lambda/D = 5.7$ and $A = 50$ V). (*b*) Stable satellite ($D = 0.27$ mm, $We = 30.6$, $\lambda/D = 5.9$ and $A = 70$ V). (*c*) Forward merge satellite ($D = 0.27$ mm, $We = 30.6$, $\lambda/D = 5.9$ and $A = 85$ V).



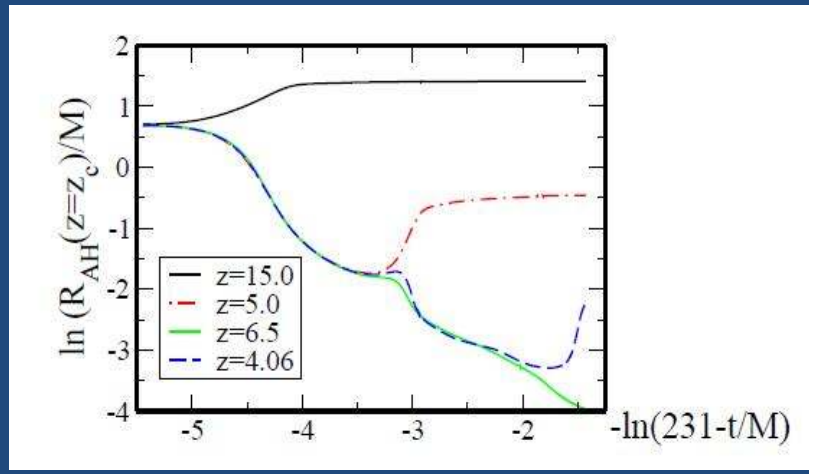
Rayleigh-Plateau instability: Satellite formation in fluids… for lower viscosity higher number of satellites [Bhatacharya etal 08, Emparan etal 10, Cardoso-Dias 06]

More than an analogy?

- Eggers, Miyamoto. In fluids, solution is self-similar, $r \sim (t_o - t)$

→ $d \ln(r) / d(-\ln(t_o - t)) = -1$

- We see roughly such slope (10-20%)



- What sets the timing?
  - 'energy' redistribution a la 'jeans instability' ?

With the right resources we can simulate situations we can't even begin to think through, and thereby provide us with ***completely new and unexpected*** *things to think about*