Band structure of the black hole entropy spectrum in LQG

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Outline

- Introduction: ‘Recalling Barbero’s talk’...
- ‘Splitting the computation’
- Two sources of degeneracy
- CFT framework
- Labeling the bands
- Conclusion

Introduction

- Quantization of an Isolated horizon in LQG (Barbero talk)
- Entropy computation reduces to a combinatorial problem: [Domagala, Lewandowski]
  - Number N(a) of sequences (m_1 , ... , m_n) satisfying:

$$\sum_{i=1}^{n} \sqrt{|m_i|(|m_i|+1)} \leq \frac{a}{8\pi\gamma\ell_P^2} \qquad \sum_{i=1}^{n} m_i = 0$$

- Solution: “Black hole generating function”

$$G(z, x_1, x_2, \ldots) = \left(1 - \sum_{i=1}^{\infty} \sum_{\alpha=1}^{\infty} (z^{k_\alpha^i} + z^{-k_\alpha^i}) x_i^{y_\alpha^i}\right)^{-1}$$

  - Pell equation
  - Combinatorics

Splitting the computation

- Solve the problem in four steps:
  - 1- Characterizing the area spectrum: Pell equation.
    - Express values of area in terms of linear combinations of ‘SRSFN’

$$a = \sum_i q_i \sqrt{p_i}, \quad q_i \in \mathbb{N} \cup \{0\}$$

    - Using Pell equation: all possible ‘spin configurations’

$$\sqrt{(k_i+1)^2 - 1} = y_i \sqrt{p_i} \quad \longrightarrow \quad x_i^2 - p_i y_i^2 = 1$$

  - 2- Compute compatible {m_i} configurations compatible with the ‘projection constraint (m-degeneracy)
    - a) Partition problem (Number theory)

$$d_m = \frac{2^N}{L} \sum_{\ell=0}^{L-1} \prod_{i=1}^{N} \cos(2\pi \ell k_i / L)$$

    - b) Introduce a generating function

$$d_m = [z^0] \prod_k (z^k + z^{-k})^{N_k}$$

Splitting the computation

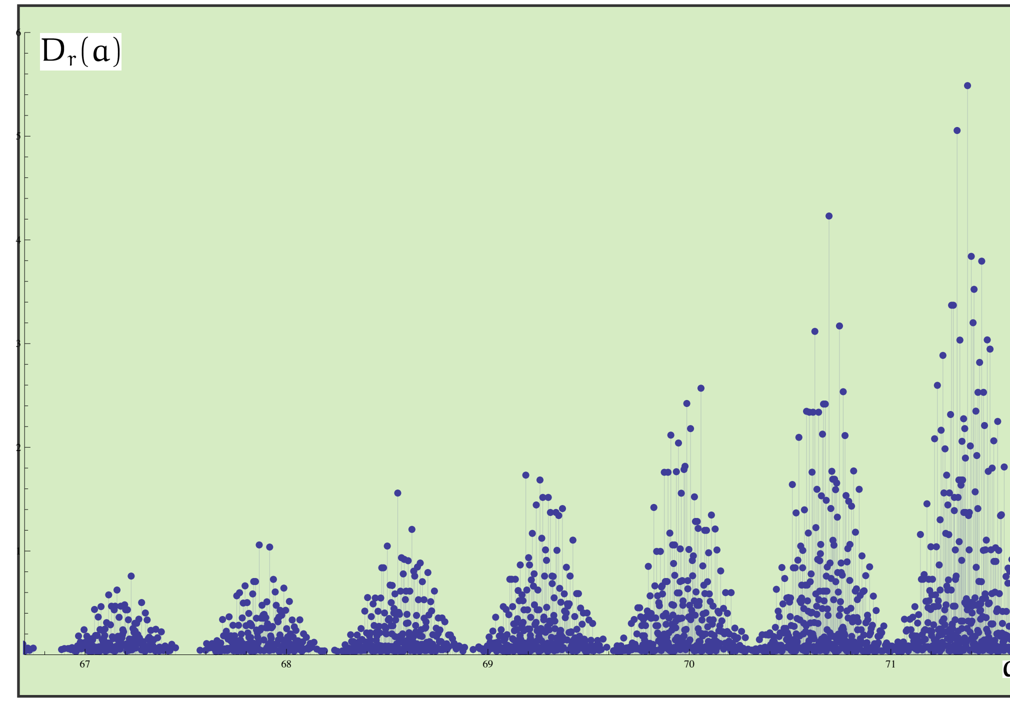
- 3- Compute all possible orderings of labels for each configuration
  - Directly related with the action of diffeomorphisms on the horizon
  - Combinatorial factor:

$$d_r = \frac{(\sum_{k=1}^{k_{max}} N_k)!}{\prod_{k=1}^{k_{max}} N_k!}$$

- 4- Sum for all areas lower than a:
  - Implement all steps together in a single generating function: 'Black hole generating function' (Barbero's talk)
  - Use Laplace-Fourier transforms to implement the sum (Barbero's talk)

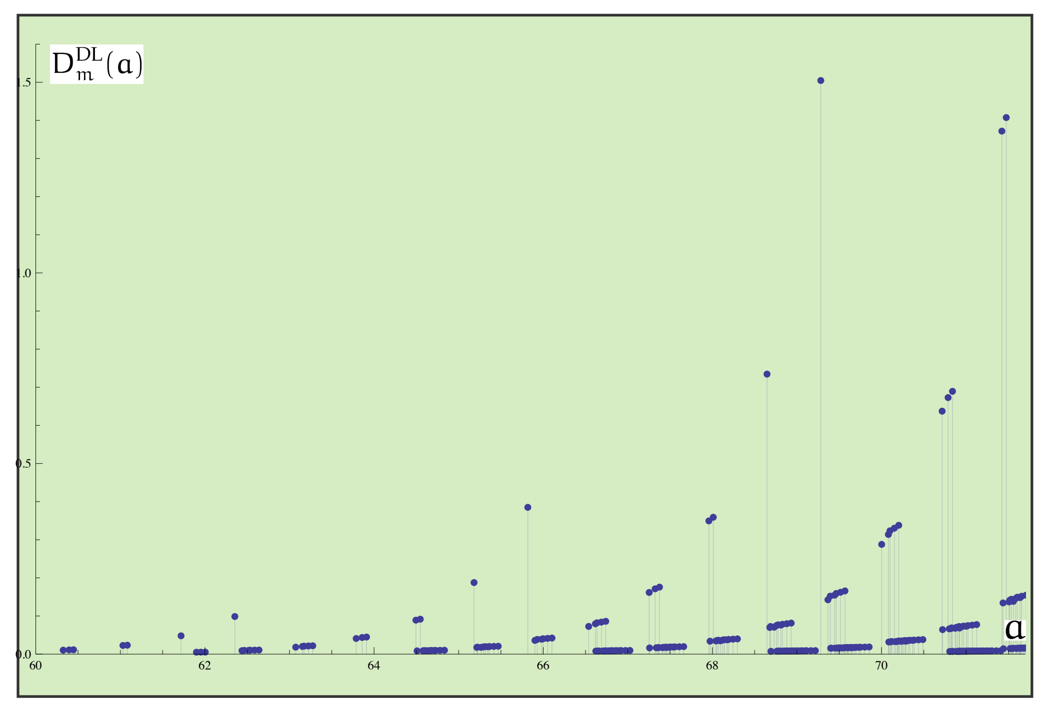
Two sources of degeneracy

- Reorderings:



  - Band structure!!!
  - They are related to diffeomorpisms

- m-degeneracy:



  - No band structure
  - Average ‘exponential’ growth
  - Some states are favored

CFT framework

- Alternative computation using some CFT-related techniques: [Agulló, Borja, DP]
  - Use analogy between SU(2) CS and WZW models
  - Restriction to U(1) at a quantum level
- Result: the m-degeneracy (same value)
- Carlip: Conformal symmetry at the heart of universality
  - Exponential growth of m-degeneracy
- The band structure is a genuine effect from LQG.

Labeling the bands

[Agullo, Borja, DP]

- How do the maximally degenerate configurations look like?

$$\hat{N}_k = \frac{N_k}{\sum_k N_k} = (k+1)e^{-\lambda\sqrt{k(k+2)}}$$

[Domagala, Lewandowski]

  - But with discrete labels only 'close' configurations to this distribution are possible
  - Special ingredient: "aproximate" constant area relation:

$$P(c) = 3\sum_k kN_k(c) + 2\sum_k N_k(c)$$

  - Maximal degeneracy only occurs for areas that correspond with integer (even) values of P(c) ⇒ Bands!

- Use P(c) to characterize bands → (See Borja's talk after the break)

Conclusion

- Black hole entropy in LQG:
  - Active field of research in the past few years
  - Consistent results
  - Specific "signature" of LQG for black hole entropy: Band structure, giving rise to the entropy quantization.
  - Many different techniques involved
    - Number theory
    - Generating functions
    - CFT related
    - Analytic combinatorics
  - Powerful methods developed, with quite a generic applicability