Quasinormal modes for the charged Vaidya metric

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Plan of the talk

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The generalized charged Vaidya metric in double null coordinates

- line element $ds^2 = -2f(u,v)dudv + r^2(u,v)d\Omega^2_{n-2}$,
- energy-momentum tensor $T_{\mu\nu} = \frac{1}{8\pi}h(u,v)k_\mu k_\nu$,
- Einstein equations

$$
\begin{aligned}
f &= 2Br_{,u} \\
r_{,v} &= -B\left(1 - \frac{2m}{(n-3)r^{n-3}} - \frac{2\Lambda r^2}{(n-2)(n-1)} + \frac{2q^2}{(n-2)(n-3)r^{2(n-3)}}\right) \\
h &= -2\left(\frac{n-2}{n-3}\right)\frac{B}{r^{n-2}}\left(\dot{m} - \frac{2q\dot{q}}{(n-2)r^{n-3}}\right)
\end{aligned}
$$

The physical picture

- Reissner Nordström black hole

$$B = -\frac{1}{2} \rightarrow \text{sets } v$$
as the proper time,

$$n = 4,$$
$$\Lambda = 0,$$
$$m = 0.5,$$
$$q = 0.4,$$

$$r(u, v=0) = -\frac{u}{2}$$

$$r_{,v} = \frac{1}{2}\left(1 - \frac{2m}{r} + \frac{q^2}{r^2}\right)$$

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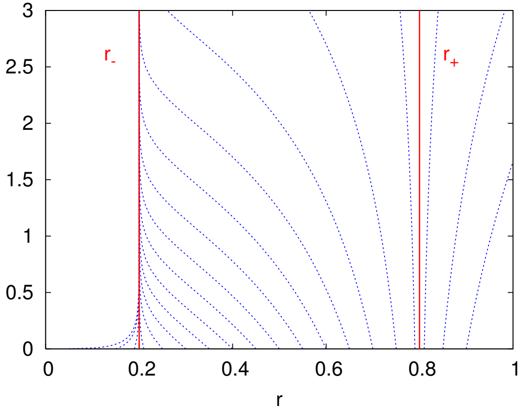

$$r_{\pm} = m \pm \sqrt{m^2 - q^2}$$

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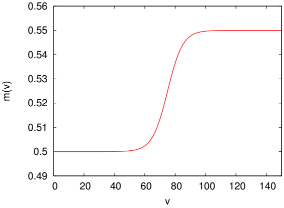
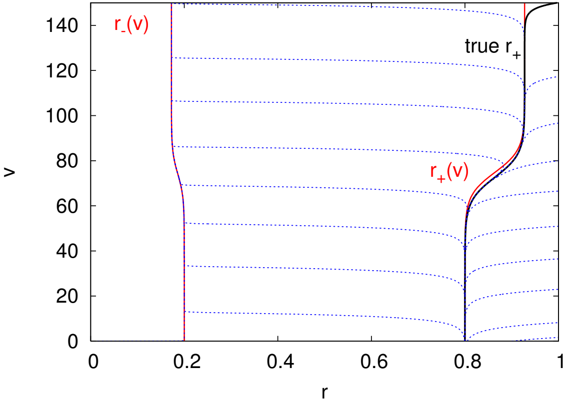
The physical picture

- Time-dependent solutions: variable mass and constant charge

$$r_{,v} = \frac{1}{2}\left(1 - \frac{2m(v)}{r} + \frac{q^2}{r^2}\right)$$

$$r_{\pm}(v) = m(v) \pm \sqrt{m^2(v) - q^2}$$





$$m(v) = \frac{m_2 - m_1}{2}\left[1 + \tanh \rho(v - v_m)\right] + m_1$$

Perturbation equations

- Klein-Gordon equation in the Vaidya background ⇒ scalar perturbation ⇒ scalar QNMs

$$\frac{1}{\sqrt{-g}}\left(\sqrt{-g}g^{\mu\nu}\Psi_{,\nu}\right)_{,\mu} = 0\,.$$

- for the general case

$$\varphi_{,uv} + V(u,v)\varphi = 0\,,$$

with the potential $V(u,v)$ given by

$$V(u,v) = \frac{f}{2}\left(\frac{\ell(\ell+n-3)}{r^2} + \frac{(n-2)(n-4)}{4r^2} + \right.$$
$$\left. + \frac{(n-2)^2 m}{2(n-3)r^{n-1}} - \frac{n\Lambda}{2(n-1)} - \frac{(3n-8)q^2}{2(n-3)r^{2(n-2)}}\right)\,.$$

- where

$$\Psi(u,v,\theta_1,\ldots,\theta_{n-2}) = \sum_{\ell,m} r^{-\frac{n-2}{2}}\varphi(u,v)Y_{\ell m}(\theta_1,\ldots,\theta_{n-2})\,.$$

Numerical integration

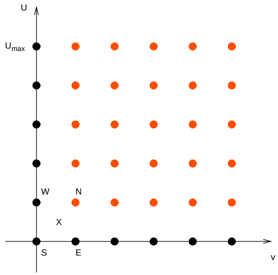
- 1st step: solve for the background to get $r(u, v)$.
- 2nd step: solve the perturbation equation on the background.

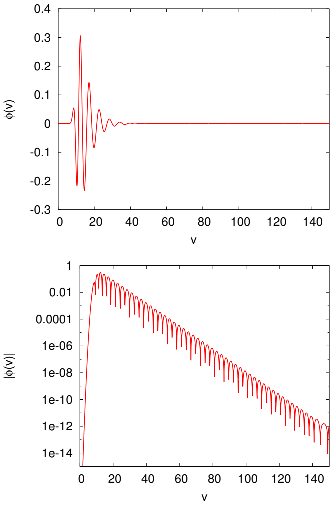
$$\varphi(N) = \varphi(W) + \varphi(E) - \varphi(S) - \Delta u \Delta v V(X) \frac{\varphi(W) + \varphi(E)}{2} + O(\Delta^4) \,.$$

- 3rd step: obtain the time-dependent scalar field on a fixed position $\varphi(v)$ and then calculate the frequencies ω_r and ω_i with a least-square fit

$$\varphi = A e^{\omega_i v} \sin(\omega_r v + \delta).$$

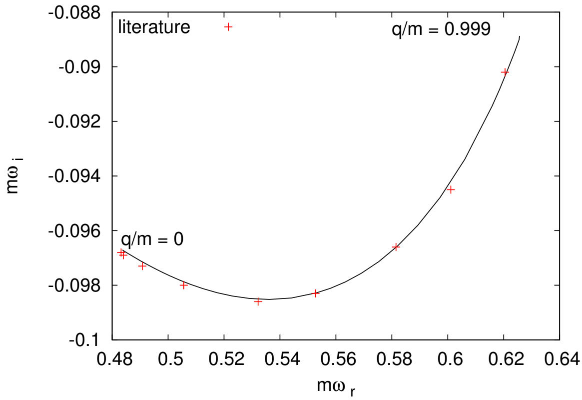
Numerical integration





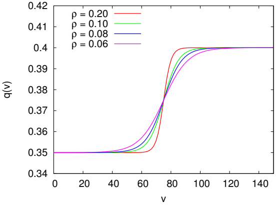
Preliminary tests

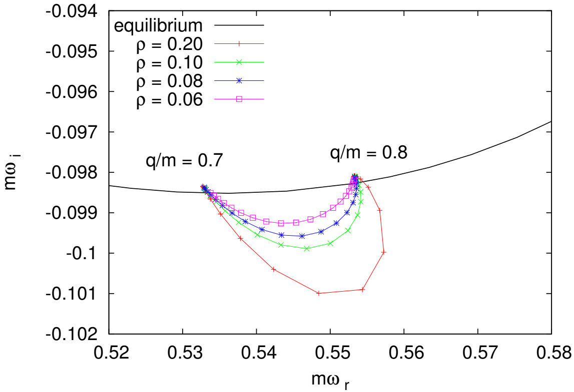
- Tests performed for Schwarzschild and RN black holes.
- QNM known values reproduced with good accuracy.



Non-adiabatic behavior

- variable charge and constant mass $m = 0.5$.
- it is possible to control how "fast" the variations are.





$$q(v) = \frac{q_2 - q_1}{2}[1 + \tanh \rho(v - v_m)] + q_1$$

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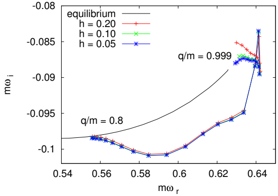
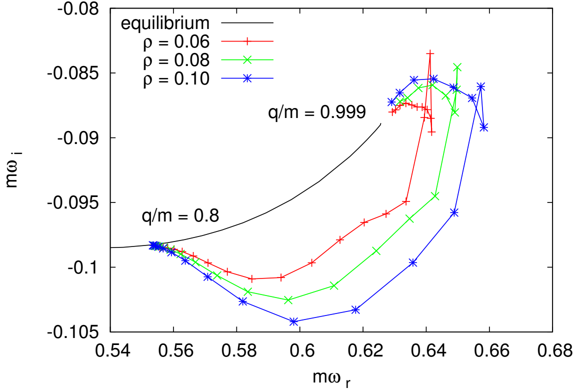
Towards an extremal RN black hole

- $m_2 \to q$
- correct asymptotic results are recovered
- do we trust these results?
  (consistency test)





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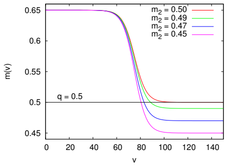
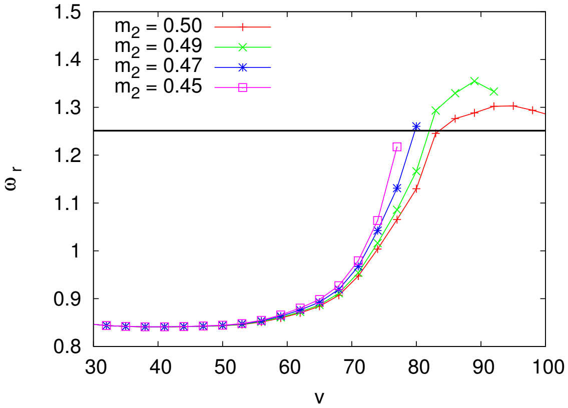
Formation of a naked singularity?

- using the choice of a final mass $m_2 < q$,
- the code crashes when $m(v) = q$, but before it crashes...





Conclusions

- Numerical setup accurately reproduces QNMs from scalar perturbations in equilibrium configurations.
- Deviations from the equilibrium frequency values were obtained and characterized.
- Non-adiabatic behavior is constrained by the threshold given by $r_+''(v) < \omega_i(v)$.
- For the future: analyze QNMs from gravitational perturbations.
- Possible observational consequences? A steadily accreting black hole could present QNMs which deviate from the equilibrium values.

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