Non-equilibrium spacetime thermodynamics, entanglement viscosity and KSS bound

goffredo chirco

SISSA - TRIESTE - ITALY

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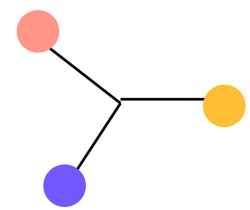
@ **ERE2010** - GRANADA

based on work with Stefano Liberati and Christopher Eling

MAIN PUZZLE

GR



THERMODYNAMICS

QFT

ANALOGY

Einstein equation --> BH mechanics <--> BH THERMODYNAMICS

IDENTITY IF ONLY + QFT:

$$S = \frac{1}{4}A$$

$$T_{Haw} = \frac{\kappa}{2\pi}$$

How did GR knows about the role of QFT ?

1968 Sakharov: GRAVITY IS INDUCED BY MATTER

given Lorentian geometry:

QFT --> geometry dynamics --> Einstein Hilbert action from one loop matter effective action

1995 Jacobson: thermodtnamical character of gravity rooted in VACUUM THERMODYNAMICS

--> EINSTEIN EQ AS AN EQUATION OF STATE

gravity not to be quantized ?

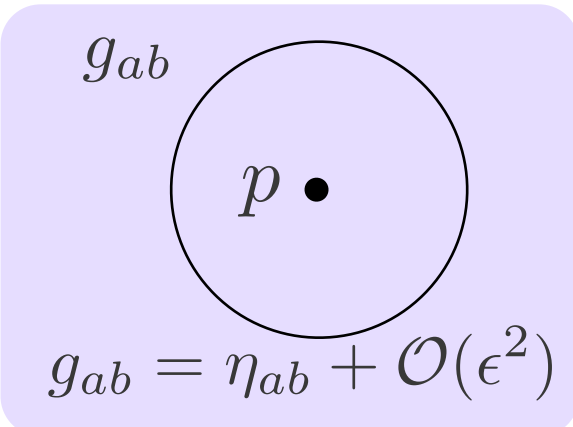
QFT as fundamental theory --> GRAVITY EMERGENT

Jacobson's perspective **HORIZON THERMODYNAMICS FROM VACUUM THERMO**

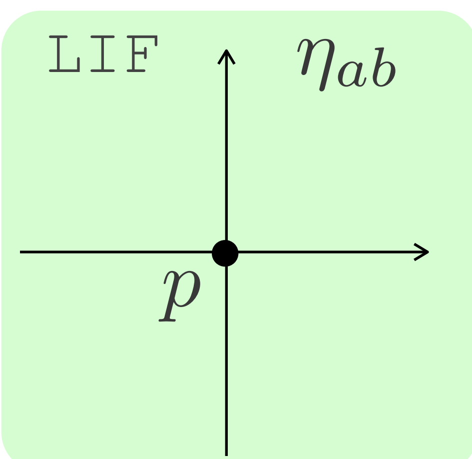
BH thermodynamics applies also to observer dependent Rindler horizons

EP everywhere in spacetime:

1 -- invoke local Lorentz invariance

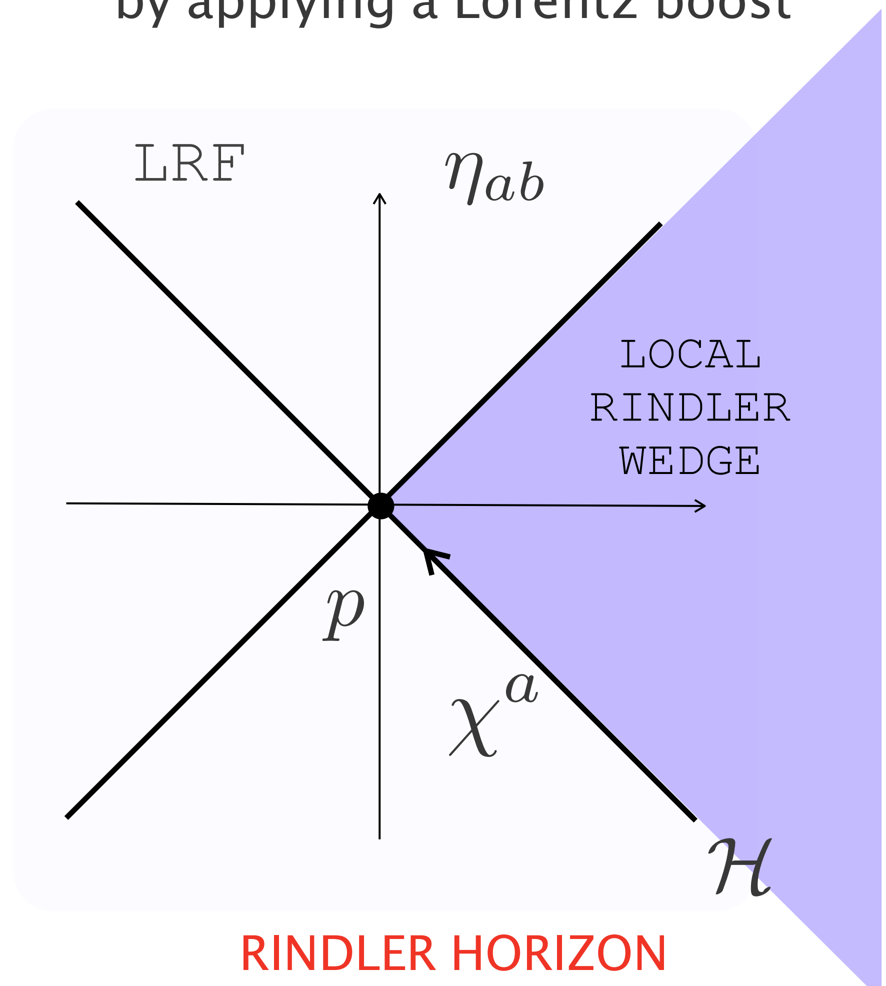


2 -- introduce a local inertial frame



where the matter fields vacuum is approximated by the Minkowski vacuum

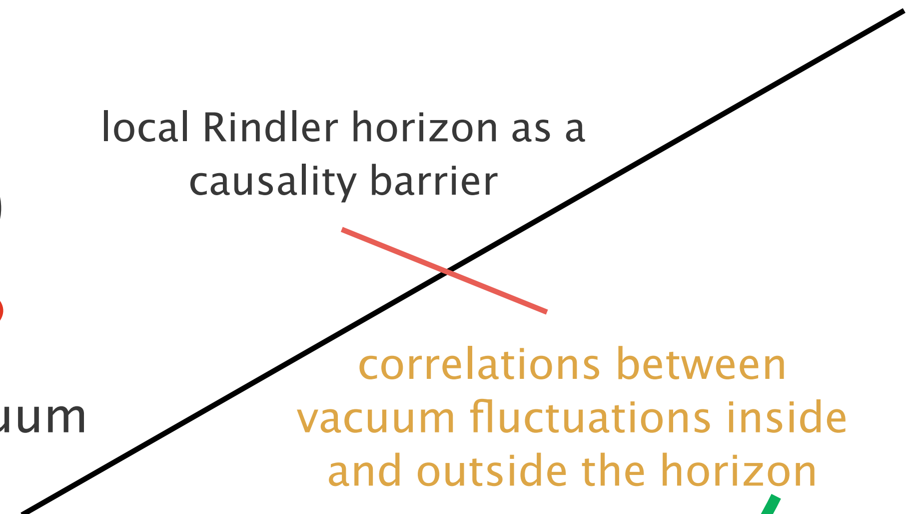
3 -- construct a local Rindler frame by applying a Lorentz boost



RINDLER HORIZON THERMODYNAMICS FROM VACUUM THERMO

- **QFT** - UNRUH EFFECT: with respect to the dimensionless boost Hamiltonian, the Minkowski vacuum is a CANONICAL ENSEMBLE

$$\rho = \exp(-\beta H)/Z \quad \Bigg| \quad \begin{array}{l} \langle E \rangle = Tr(\rho H) \\ S = -Tr(\rho ln \rho) \end{array}$$



- **SM** - ENTANGLEMENT ENTROPY of vacuum

- **HOLOGRAPHY** - ent entropy scales with the horizon surface area + UV cutoff --> $S = \alpha A$

for $\delta\rho \ll \rho$ --> $\delta S = \beta \delta \langle E \rangle$

EQUILIBRIUM

- $\delta \langle E \rangle = \int T_{ab} \chi^a d\Sigma^b$ energy crossing the horizon = vacuum perturbation

--> --> $\delta \langle E \rangle = \delta(\alpha A)$

LOCAL MATTER-GEOMETRY CONSTITUTIVE RELATION

DETAILS AND GENERALIZATION TO NONEQUILIBRIUM THERMO

$S = \alpha A$ — α depends on the nature of the quantum fields and their interactions and can be some complicate function of the position in spacetime

- assume $\alpha = const$ --> $\delta S = \alpha \delta A$

$\delta A = \int_H \tilde{\epsilon}\, \theta\, d\lambda$ where $\theta \approx \theta_p + \lambda \left.\frac{d\theta}{d\lambda}\right|_p + \mathcal{O}(\lambda^2)$

by Raychaudhuri eq. $\frac{d\theta}{d\lambda} = -\frac{1}{2}\theta^2 - \|\sigma\|^2 - R_{ab}l^a l^b$ PURELY GEOMETRIC

$$dS = \alpha \int_H \tilde{\epsilon}\, d\lambda [\theta - \lambda(1/2\,\theta^2 + \|\sigma\|^2 + R_{ab}l^a\, l^b)]_p$$

REVERSIBLE LEVEL

$$TdS = \alpha \frac{\kappa\hbar}{2\pi} \int_H \tilde{\epsilon}\, d\lambda [\theta - \lambda(1/2\,\theta^2 + \|\sigma\|^2 + R_{ab}l^a\, l^b)]_p =$$

unmatched terms associated to purely geometrical DOF

$$= \int_H \tilde{\epsilon}\, d\lambda\, (-\lambda\kappa) T_{ab} l^a l^b = \delta Q$$

heat current linear in λ --> θ

- interpretative guess: non-equilibrium thermodynamics

$$dS = dS_{eq} + dS_i$$

IRREVERSIBLE LEVEL internal entropy production

NON-EQUILIBRIUM THERMODYNAMICS --> VISCOSITY

REVERSIBLE LEVEL: $\frac{2\pi}{\hbar\alpha} T_{ab} = R_{ab} + \Phi\, g_{ab}$ for all null l^a

define $\Phi = -\frac{1}{2}R - \Lambda$ by

- local energy conservation $\nabla^b T_{ab} = 0$
- Bianchi identity $\nabla^b R_{ab} = \frac{1}{2}\nabla_a R$

then GR $8\pi G\, T_{ab} = R_{ab} - \frac{1}{2} R\, g_{ab} - \Lambda\, g_{ab}$ if only $\alpha = \frac{1}{4G}$

EP everywhere in spacetime

IRREVERSIBLE LEVEL: $dS_i = -\alpha \int_H \tilde{\epsilon} d\lambda \lambda \|\sigma^2\|$

UV cutoff--> G
INDUCED GRAVITY
G CONSTANT -- SEP

- membrane paradigme: horizon dynamics is well described by 2+1 viscous fluid equations

$$ds_i = \frac{\zeta}{T}(\nabla^c v_c)^2 + \frac{2\eta}{T}\|\sigma\|^2$$

--> $\eta = \frac{T\alpha}{2} = \frac{\hbar\alpha}{4\pi} = \frac{1}{16\pi G}$ horizon viscosity coefficient --> associated dissipated energy ?

YES! $T dS_i = 2\eta \int_H \tilde{\epsilon} dv \|\sigma^2\| = \frac{1}{8\pi G}\int_H \tilde{\epsilon} dv \|\sigma^2\|$ same as BH TIDAL HEATING

FOCUSSING ON THE IRREVERSIBLE SECTOR...

thermodynamical approach goes beyond the reversible equation of motion (action)

-- GR irreversible sector contains the information about the propagation of gravitational degrees of freedom (GR-tensorial, f(R)-scalar+tensorial)

-- dissipation at macro level still regulated by phenomenological viscosity coefficients defined by the UV cutoff --> vacuum fluctuation about thermal eq

MORE..

$$\eta/\alpha = \frac{1}{4\pi}$$

as for BH, the shear viscosity to entropy density ratio for the Rindler horizon sistem saturate the Kotvun Son Starinets (KSS) bound

KSS ratio seem to be rooted in gravitational physics, but the Rindler wedge is a subregion of Minkowski space time: NO GRAVITY AT ALL

INTERESTING...

if one could calculate η directly from the fluctuations of the matter fields in the thermal vacuum, that would characterize the KSS bound as a fundamental property of quantum entanglement and its associated holography

AN ENTANGLEMENT VISCOSITY...

PROBLEM

how to relate a phenomenological transport coefficient η from a fluid-wise description (membrane) of the horizon to the quantum vacuum state on the bulk ??

no holographic duality like AdS/CFT in Rindler wedge

... but PRE-HOLOGRAPHY

area scaling behaviour of entanglement entropy: quantum degrees of freedom of the wedge seem to be packed on the stretched horizon surface	-->	TRY a lower dimensional description of the vacuum fields associated to the horizon

MAIN IDEA

(AdS/CFT)

- on large spatial and time scale the thermal vacuum can be effectively described by hydrodynamics
- calculate the hydrodynamics transport coefficient from microscopic theory using KUBO FORMULA involving the Green's function of the energy momentum ST for the matter fields in the wedge

LOWER DIMENSIONAL DESCRIPTION OF BULK FIELDS

WHAT WE WANT:

dual lower dimensional description of the vacuum state in terms of a strongly coupled thermal CFT living effectively on a (D-1) Minkowski (horizon membrane)

RECEIPT

- start with the canonical energy momentum tensor for the Rindler wedge

$$T_{(R)}{}^{\mu}_{\nu} = \frac{\partial L_R}{\partial(\partial_\mu)\psi}\partial_\nu\psi - \delta^\mu_\nu L_R \qquad L_R = \sqrt{-g}L_M$$

- ANSATZ: on large scales, the holographic vacuum state is described by a conserved lower dimensional SET

$$\langle\hat{T}^{(D-1)}_{\mu\nu}\rangle = Z^{-1}Tr(\rho\,\hat{T}^{(D-1)}_{\mu\nu}) = \langle 0|\hat{T}^{(D-1)}_{\mu\nu}|0\rangle$$

Minkowski vacuum expectation value

thermal average at Tolman-Unruh temperature $\quad T_{(R)}{}^{\mu}_{\nu} = \kappa\xi T_{(M)}{}^{\mu}_{\nu}$

- DIMENSIONAL REDUCTION

$$\langle\hat{T}^{(D-1)}_{\mu\nu}\rangle = \int_{lc}^{\infty} d\xi\langle\hat{T}^{(R)}_{\mu\nu}\rangle = \int_{lc}^{\infty} d\xi\,\kappa\xi\langle\hat{T}^{(M)}_{\mu\nu}\rangle$$

A KUBO LIKE FORMULA FOR THE HORIZON VISCOSITY

II apply the formalism of viscous hydrodynamics and calculate the shear viscosity through a Green-Kubo approach in terms of the effective lower dimensional SET

- consider a metric perturbation $h_{\mu\nu}$ associated to the bulk vacuum perturbation $\delta\langle E\rangle$ as source for the (D-1) field theory operator $\hat{T}^{D-1}_{\mu\nu}$

- assuming the perturbation is small, from linear response theory, one can calculate the change of the expectation value of $\hat{T}^{D-1}_{\mu\nu}$

$$< \delta\hat{T}^{D-1}_{\mu\nu}(k^0,\vec{k}) >= G_R(k^0,\vec{k})\, h_{\mu\nu}(k^0,\vec{k})$$

where G_R is the retarded 2-point thermal Green's function of $\hat{T}^{D-1}_{\mu\nu}$

$$G_R(k^0,\vec{k}) = \int d\tau d^{D-2}x\, e^{ik^0t}\, e^{-i\vec{k}\vec{x}} < [\hat{T}^{D-1}_{\mu\nu}(x)\hat{T}^{D-1}_{\mu\nu}(0)] >$$

- in the limit $(k^0,\vec{k}) \to 0$ $\quad < \hat{T}^{D-1}_{xy}, \hat{T}^{D-1}_{xy} > (k^0,\vec{k}\to 0) = i\eta k^0 - P + \mathcal{O}(\omega^2)$

from which one gets the quantitative expression for the shear viscosity

$$\eta = \lim_{k^0\to 0} \frac{1}{k^0}\, G_R^{xy,xy}(k^0,0)$$

KSS BOUND FOR THE RINDLER HORIZON

- in our particular case we have

$$\eta = \lim_{k^0 \to 0} \frac{1}{k^0} \int_{l_c}^{\infty} \xi' \int_{l_c}^{\infty} \xi \int d\tau d^{D-2}x \, e^{ik^0\tau} \theta(\tau) \kappa^2 \xi \xi' < [T_{xy}^D(\tau, x, y, \xi), T_{xy}^D(0, \xi')] >$$

where $< [T_{xy}^D(\tau, x, y, \xi), T_{xy}^D(0, \xi')] >$ is the Minkowski 2-point correlator of the bulk field theory

THEN

for a free minimally coupled scalar field in 4D Rindler spacetime is

$$\eta = \frac{1}{1440\pi^2 l_c^2}$$

AREA SCALING ENTANGLEMENT VISCOSITY

- from the thermal description of the dimensionally reduced vacuum fields

$$\epsilon = \frac{\pi^2 T^4}{30} = \frac{1}{480\pi^2 \xi^4} \quad \text{--->} \quad \epsilon_r = \frac{\kappa}{960\pi^2 l_c^2}$$

$$s = \frac{2\pi^2}{45} T^3 = \frac{1}{180\pi \xi^3} \quad \text{--->} \quad s = \frac{1}{360\pi l_c^2}$$

all area scaling quantities

4D - Planckian form : massless free scalar field = ultrarelativistic boson gas $\epsilon = 3P$

---> $\eta/s = \frac{1}{4\pi}$ KSS bound satisfied by just entanglement quantities !!!

RESULTS AND PERSPECTIVES...

NON equilibrium thermodynamical description:

propagarion of purely gravitational dof associate with macro dissipative effects

gravity field equations:
equililibrium dynamics
(reversible) ---/--> spacetime intrinsically non-dissipative!!

KSS ratio from entanglement:

local Rindler horizon system | NO GRAVITY
| NO HOLOGRAPHIC DUALITY LIKE ADS/CFT

GOAL to find a microscopic description for the shear viscosity in terms of the fluctuations of the Rindler wedge thermal state in a finite temperature QFT

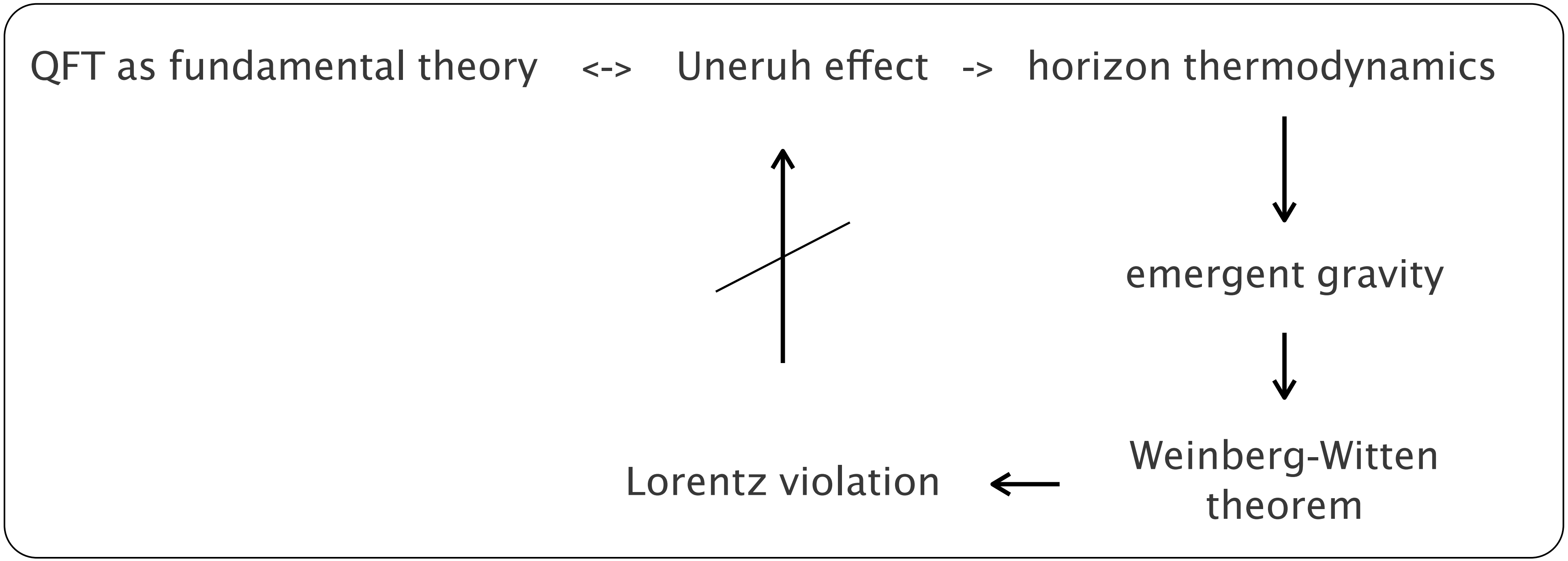
---> KSS ratio may be a fundamental holographic property of spacetime and quantum entanglement

support for the hypothesis that semi-classical gravity on macroscopic scales is induced or emergent as an effective theory of some lower dimensional strongly coupled quantum system with a large number of degrees of freedom

THANK YOU!

OPEN ISSUES...

In a deeper emergenr gravity perspective :



HOLOGRAPHY: dimensionally reduced bulk QFT living on the horizon screen maybe allows to pass over W-W ?

BEYOND GR...?

- SEP fixes G to be a universal constant --> no extra gravitational fields

RESULT GR irreversible sector contains the information about the propagation of tensorial gravitational degrees of freedom

- relaxing SEP to EEP one can consider more general theories of gravity

G is now spacetime location dependent and it is promoted to a spacetime field which need to be dynamical to assure the background independence of the resulting theory

GUESS extra dynamical propagating degrees of freedom are still associated with the irreversible sector of the theory

SIMPLEST EXAMPLE

f(R)

$S = \alpha F(R)\tilde{\epsilon}$ --> $dS = \alpha \int_H \tilde{\epsilon}\, d\lambda\, (\dot{F} + F\,\theta)$

$dS = 0$ --> $\theta_p = -\dot{F}/F \neq 0$

extra contribution coupled to the dynamics of the scalar function F

$$F R_{ab} - F_{;ab} + (\Box F - 1/2 f) g_{ab} = 2\pi/\hbar\alpha T_{ab}$$

$$\delta N = -\int_H \tilde{\epsilon}\, d\lambda\, \lambda(\alpha F)[3/2\theta^2 + \|\sigma^2]_p$$

internal entropy production: now TWO contributions from scalar and tensorial degrees of freedom. EEP allows for scalar d.o.f.

A KUBO LIKE FORMULA FOR THE HORIZON VISCOSITY

- in the large scale limit , $(k^0, \vec{k}) \longrightarrow 0$, the perturbed vacuum state is characterized by a dimensionally reduced SET in the viscous fluid form

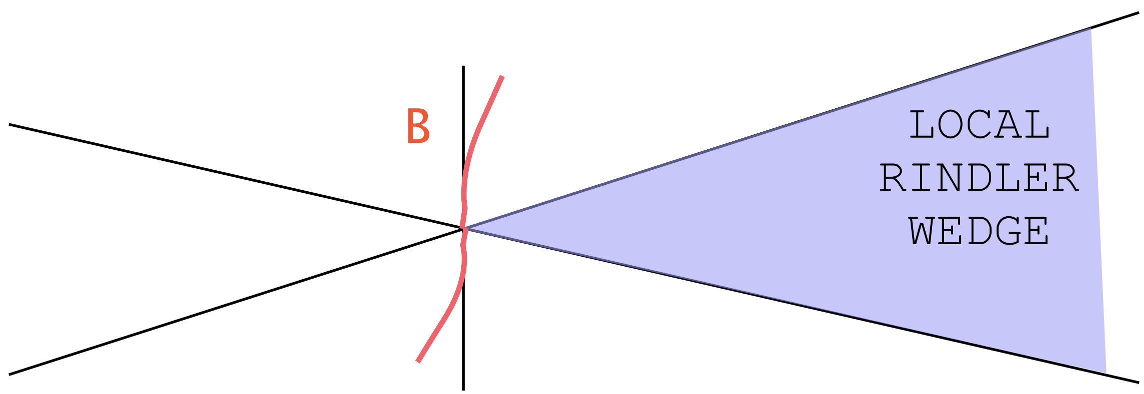
$$< T^{\mu\nu}_{D-1} >= (\epsilon + P)u^\mu u^\nu + P g^{\mu\nu} - \eta P^{\mu\alpha} P^{\nu\beta} \left(\nabla_\alpha u^\beta + \nabla_\beta u^\alpha - \frac{2}{D-1} g_{\alpha\beta} \nabla \cdot u \right)$$

equilibrium-perfect fluid

non-equilibrium-viscous stress

-- the metric perturbation $h_{\mu\nu}$ lives in the $(\tau, \xi = 0)$ plane (D-2 B surface bending)

---> just $h_{xy} \neq 0$



-- therefore one is interested only in the -xy- component of the perturbed SET

$$< T^{D-1}_{xy} >= P\, g_{xy} - \eta(\nabla_x u_y + \nabla_y u_x) = P\, h_{xy} + \eta\, \partial_0 h_{xy} = \int d^{D-1}x'\, G^{ij}_R(x, x')\, h_{xy}(x')$$