Black holes die hard: Can one spin up a black hole past extremality?

Mariam Bouhmadi-López
mariam.bouhmadi@ist.utl.pt

CENTRA, Instituto Superior Técnico
(Lisbon, Portugal)

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in collaboration with Vitor Cardoso, Andrea Nerozzi and Jorge V. Rocha

Black holes and naked singularities

- Black Holes can be formed through (e.g.)
  - The collapse of matter
  - Sufficiently high energy collisions of particles
  - Quantum fluctuations in the early universe
- Any process capable of confining a large portion of matter in a small enough space leads to the formation of trapped surfaces.
- In "principle", these singularities need not be hidden by an event horizon. Therefore, naked singularities could arise.

Cosmic Censorship

- The existence of naked singularities would represent a break down of causality and would lead to a break down of predictability of general relativity.
- In any case singularities are expected to arise (Penrose and Hawking singularity theorems).
- The (weak) Cosmic Censorship Conjecture proposed by Penrose in 1969 is a way to “avoid” the problem: Generically, any singularities present in the end point of a gravitational collapse of physically acceptable matter must be hidden within black holes.
- So far, the Cosmic Censorship Conjecture has not been (dis)-proved.

Black holes are hard to kill

- No classical mechanism has been found that can destroy a black hole and therefore violates the Cosmic Censorship Conjecture.
- The collision at close to the speed of light of two equal-mass black holes with an arbitrary impact parameter lead to a Kerr black hole. Therefore no naked singularity is formed (Sperhake et al, PRL 103, 131102 (2009)).
- A notable attempt to destroy a BH was suggested by Wald in 1974 but the mechanism was shown simultaneously to be unsuccessful (Wald, Ann. Phys. 82, 548 (1974)).
- Over-spinning a black hole with a test body was suggested recently, but backreaction and self-forces effects were neglected, which may prevent the over-spinning (Jacobson and Sotiriou, PRL 103, 141101 (2009)).

Wald's Gedanken Experiment to destroy a BH

- Wald experiment consisted in throwing a point particle at a (four-dimensional) Kerr black hole with just the right angular momentum to spin the black hole up in such a way that eventually the horizon is disrupted (Wald, Ann. Phys. 82, 548 (1974)).
- The angular momentum of Kerr black holes is bounded $J \leq M^2$, thus *if it were possible* for the black hole to capture particles of high enough angular momenta, then one might exceed this bound, possibly creating a naked singularity.
- We remind that for $J \leq M^2$ there is a black hole with $J = M^2$ corresponding the extremal case. On the other hand, $J > M^2$ there is a naked singularity.
- Wald showed this cannot happen, as the potentially dangerous particles (i.e., those with large enough angular momentum) are never captured by the black hole.

Purpose/objective

- The purpose is to extend Wald's analysis to other spacetimes: the Myers-Perry family of rotating black holes in higher dimensions.
- This analysis is interesting because it allows one to test Cosmic Censorship in a very simple, yet realistic scenario.
- The results obtained with "point-particles" could be used as a guidance to numerical studies of Black holes collisions in four- and even the on-going efforts in higher dimensions (E. Berti, V. Cardoso, T. Hinderer, M. Lemos, F. Pretorius, U. Sperhake & N. Yunes, 2010, Zilhão, Witek, Sperhake, Cardoso, Gualtieri, Herdeiro & Nerozzi, 2010).

Myers-Perry family rotating black holes in higher dimensions

- A (asymptotically flat) stationary BH is described by the Myers-Perry solution.
- In four dimensions, there is only one possible rotation axis for a cylindrically symmetric spacetime, and there is therefore only one angular momentum parameter.
- In higher dimensions; $5 \leq D$; there are several choices of rotation axis and there is a multitude of angular momentum parameters, each referring to a particular rotation plane. The general solution contains $[(D-1)/2]$ parameters related to the angular momenta.
- In 5D the angular momenta are bounded by $(|a_1| + |a_2|)^2 \leq M$.
- Singly-spinning MP in $6 \leq D$ has no upper limit on its angular momentum.
- When all angular momenta are equal there is an upper bound, in any D.

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Test-article in the equatorial plane

- We consider the test-particle approximation where backreaction is neglected.
- We focus exclusively on the intuitively most dangerous process: particles falling in along the equator. We will also consider two special sub-cases of the geometries discussed so far, (i) black holes with a single rotation parameter and (ii) black holes with all rotation parameter equal.
- The motion of the test particle along the equatorial plane can be described using the effective "2+1" metric:
  - The test particle motions is described by the geodesic: $g_{\mu\nu}\dot{x}^\mu\dot{x}^\nu = -\delta_1$, where $\delta_1 = 1, 0$ for timelike and null geodesic .
  - The symmetries of the background determine conserved quantities $E \equiv -g_{\mu\nu}(\partial/\partial t)^\mu\dot{x}^\nu \, , \qquad L \equiv g_{\mu\nu}(\partial/\partial\psi)^\mu\dot{x}^\nu \, ,$
  - The "radial" motion of the test particle follows from the previous relation: $\dot{r}^2 = V_r$

Spinning-up a black hole by throwing point particles-1-

- We try to spin-up a BH with mass $\mathcal{M}_0$ and angular momentum $\mathcal{J}_0$ in general D spacetime dimensions. For that, we throw in a particle of mass m_0 with angular momentum $\delta\mathcal{J} = m_0 L$ and energy $\delta\mathcal{M} = m_0 E$, such that $\delta\mathcal{M} \ll \mathcal{M}_0$ and $\delta\mathcal{J} \ll \mathcal{J}_0$.
- The Myers-Perry BH is characterised by parameters M and a_i , associated to the mass and angular momenta:

$$\mathcal{M} = \frac{D-2}{16\pi} A_{(D-2)} M, \quad \mathcal{J}_i = \frac{1}{8\pi} A_{(D-2)} M a_i, \quad (i = 1, ..., [(D-1)/2])$$

where $A_{(D-2)}$ is the area of a unit $(D-2)$-sphere

Spinning-up a black hole by throwing point particles-2-

- The dimensionless spin of the BH

$$j \equiv \frac{\mathcal{J}}{\mathcal{M}^{\frac{D-2}{D-3}}}$$

- Upon absorption of this particle, the dimensionless spin of the BH

$$j = j_0 + \delta j$$

where the subscript stands for initial parameters of the BH and

$$\delta j = \frac{m_0}{\mathcal{M}_0}\left(\frac{L}{\mathcal{M}_0^{\frac{1}{D-3}}} - E j_0 \frac{D-2}{D-3}\right)$$

Singly spinning Myers-Perry BH -1-

- Equatorial motion can be reduced to the following radial equation

$$\dot{r}^2 = V_r \quad r^2 V_r = \left[r^2 E^2 + \frac{M}{r^{D-3}}(aE - L)^2 + (a^2 E^2 - L^2) - \delta_1 \Delta_D \right] .$$

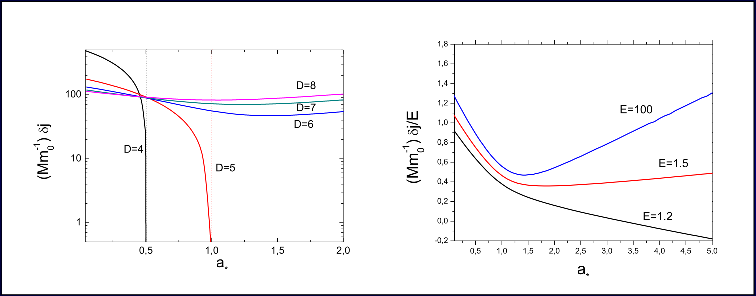
- The radial motion is completely governed by the potential V_r. If there are turning points outside the event horizon, then a particle coming from infinity can not reach the event horizon. Thus, the analysis we want to make is to study the maximum value of L, $L_{\rm crit}$, for which there are either no turning points, or all of them lie inside the event horizon.
- For $D = 5$: $\quad L_{\rm crit} = E\sqrt{M} + \sqrt{E^2 - 1}\left(\sqrt{M} - a\right)$

$$\Longrightarrow (\delta j)_{\rm max} = \frac{m_0}{\mathcal{M}_0^{3/2}} \left(\sqrt{M} - a\right)\left(E + \sqrt{E^2 - 1}\right)$$

- Cannot spin-up the BH above the extremal limit in 5D

Singly spinning Myers-Perry BH -2-

- What about higher dimensions? For larger D, there is no extremal limit, and the black holes can be spun-up to an arbitrarily high rotation.



- The variation in the dimensionless spin depends sensitively on E: (i) $\exists\ E_{\rm crit}$ above which $\delta j_{\rm max}$ is a growing function of the dimensionless rotation parameter $a_* \equiv \frac{a}{M^{1/(D-3)}}$ (at large a_*), while for values of $E < E_{\rm crit}$, $\delta j_{\rm max}$ is a decreasing function of $a_* \equiv \frac{a}{M^{1/(D-3)}}$. (ii) $E_{\rm crit}$ gets smaller as D increases.

Spinning Myers-Perry BH with equal a_i -1-

- Equatorial motion can be reduced to the following radial equation:
  - Even D:

$$r^3 V_r = M\left(r^2+a^2\right)^{(4-D)/2}\left[r^2\delta_1 + (L-aE)^2\right] + r\left[(r^2+a^2)(E^2-\delta_1) - L^2\right]$$

  - Odd D:

$$r^2 V_r = M\left(r^2+a^2\right)^{(3-D)/2}\left[r^2\delta_1 + (L-aE)^2\right] + \left[(r^2+a^2)(E^2-\delta_1) - L^2\right]$$

- When $a = a_i$, all the spin of the BH are bounded $a \leq M^{1/(D-3)}/2$.
- As we are dealing with a MP BH with equal a_i, by throwing only 1 point particle, we would end up with a MP BH with different a_i where the previous bound does not apply and we could conclude erroneously a violation of the cosmic censorship (example $D = 5$).

Spinning Myers-Perry BH with equal a_i -2-

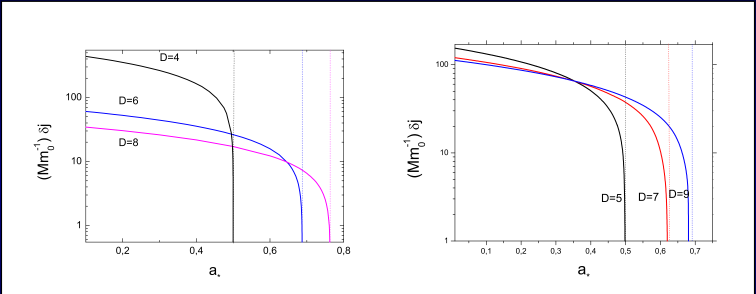
- How can we avoid this situation? Instead of throwing in one test particle, consider d particles following similar geodesics along the d orthogonal rotation planes. The final black hole will also have all angular momenta equal.
- For D=5 (the extremal limit, $a \to \sqrt{M}/2$)

$$(\delta j_1)_{\max} = (\delta j_2)_{\max} = \frac{m_0}{\mathcal{M}_0^{3/2}} \left[E\left(\sqrt{M} - 2a\right) + \sqrt{(E^2-1)\left(M - 2a\sqrt{M}\right)} \right]$$

- The same *rationale* can be applied for higher dimensions; i.e.

$$\delta j = \frac{m_0}{\mathcal{M}_0} \left(\frac{L}{\mathcal{M}_0^{\frac{1}{D-3}}} - d\, E j_0 \frac{D-2}{D-3} \right)$$

Spinning Myers-Perry BH with equal a_i -3-



Cannot spin-up the BH above the extremal limit

Conclusions

- We have shown that several different black hole geometries are immune to the throwing of point particles (in the geodesic approximation employed).
- Particles which are captured by the black hole have an angular momentum which is sufficiently low so as to be harmless; in fact sufficiently low that they are never able to spin-up the geometry past the extremal value.