Non-linear preheating after inflation and gravitational wave production

- Inflation → (p)reheating
- Preheating:
  - Field + metric perturbations
  - Preheating of tensor modes: Hybrid models
- Summary

Mar Bastero Gil
University of Granada

MBG, M. Tristram,
J. Macias-Perez, D. Santos PRD77 '08
MBG, J. Macías-Pérez, D. Santos, PRL105 '10

Expanding Universe

Flatness problem

$\Omega_T = 1 \Rightarrow \Omega_T(t_{nucl}) - 1 \approx 10^{-16}$

Horizon problem

The observable Universe was larger than the **particle horizon** at LSS

Inflation

Early period of accelerated expansion

$\ddot{a} > 0: \quad P < -\rho/3$

Super-horizon perturbations?

Too small sub-horizon (**causal**) perturbations

Unwanted relics

monopoles, moduli, gravitinos,...

Starobinsky ‘80; Guth ‘81; Albrecht, Steinhardt ‘82; Linde '82

Slow Roll Inflation

Scalar field rolling down its (flat) potential

- Quantum fluctuations → <u>Primordial Spectrum</u>

$$P_R^{1/2} \simeq (H_k/\dot{\phi})P_{\delta\phi}^{1/2} \simeq 5\times 10^{-5} \qquad (k \simeq a_0 H_0)$$

(Scalar)

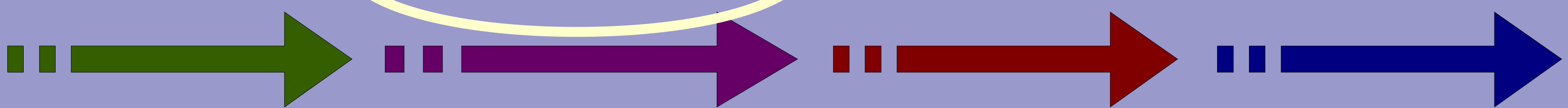
(Superhorizon)

- Primordial <u>Gravitational Waves</u>:

$$ds^2 = dt^2 - a(t)^2(\delta_{ij}(1+2\psi)+h_{ij})dx^i dx^j \qquad P_T^{1/2} \sim 2\sqrt{2}H/(2\pi m_P)$$

Tensor-to-scalar ratio r <1

Inflation (P)Reheating Radiation Matter



Oscillations : Matter $\dot{\rho}_\phi + 3H\rho_\phi + \Gamma_\phi \rho_\phi = 0$

Radiation $\dot{\rho}_R + 4H\rho_R - \Gamma_\phi \rho_\phi = 0$

<u>The inflaton interacts with other fields</u>

Preheating

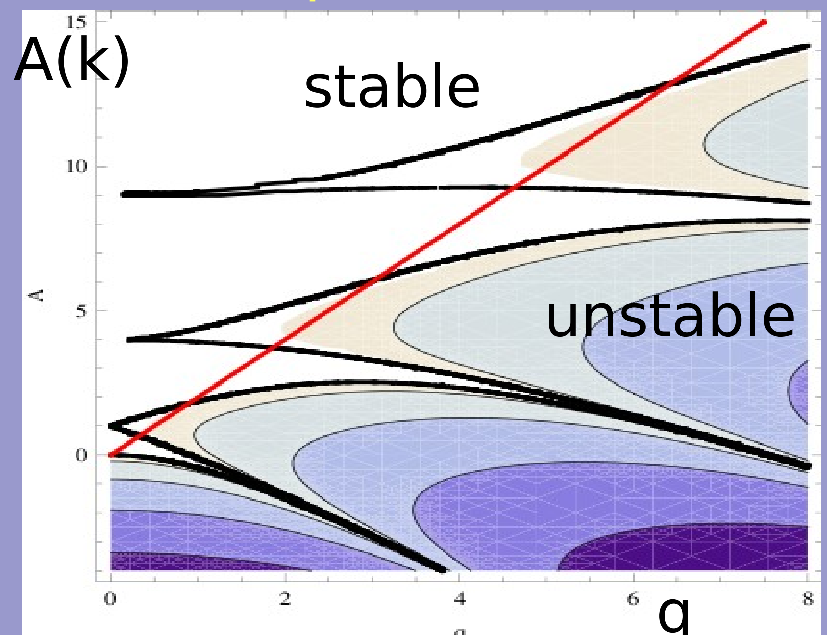
$$V = V(\phi) + \frac{g^2}{2}\phi^2\chi^2 \quad \Rightarrow \quad m_\chi^2(t) = g^2\phi^2(t)$$

- Non-adiabatic change in the time dependent effective masses leads to parametric amplification of the field fluctuations

$$\delta\ddot{\chi}_k + 3H\delta\dot{\chi}_k + \left(k^2/a^2 + m_\chi^2(t)\right)\delta\chi_k = 0$$

$$\dot{m}_\chi(t)/m_\chi^2(t) \gg 1 \quad \Rightarrow \quad \delta\chi_k \sim \exp(2\mu_k\omega t)$$

Particle production within certain resonance bands in k-space



- Mathieu equation:

$$\psi_k'' + (A(k) - 2q\cos 2z)\psi = 0$$

$$(\psi_k = a^{3/2}\delta\phi, \;\; z = \omega t)$$

$A(k) = 2q + k^2/\omega^2 a^2$

$q = \langle m^2(t) \rangle / 4\omega^2$

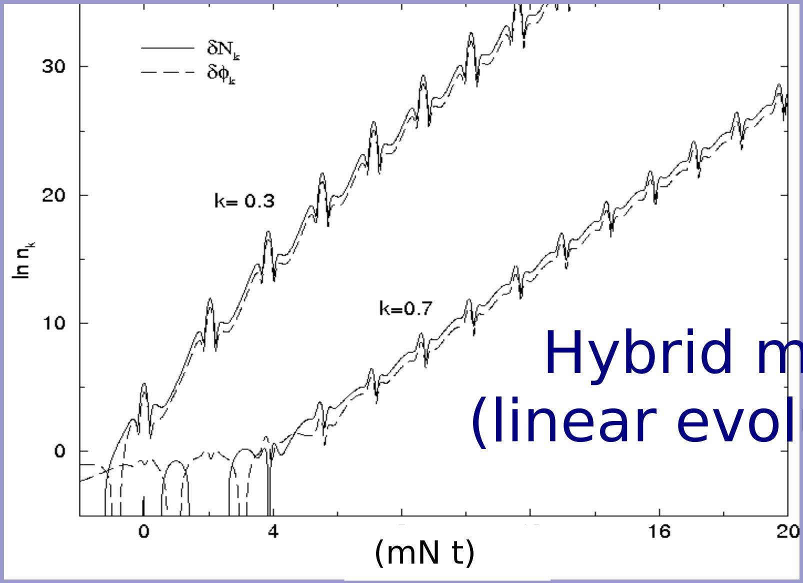
$q<1$: narrow resonance

$q>>$: broad resonance

Dolgov & Kirilova '90;
Traschen & Brandenberger '91; Kofman, Linde, Starobinsky '94, '97

Preheating

- Linear perturbation theory breaks down when



Hybrid model
(linear evolution)

$$\langle (\nabla \chi)^2 \rangle \sim \langle \dot{\phi}^2 \rangle \sim V(\phi)$$

Gradient ~ Kinetic ~ Potential

$$\ddot{\chi}(x) + 3H\dot{\chi}(x) - \frac{1}{a^2}\nabla^2 \chi(x) + \underline{g^2 \chi(x) \phi(x)^2} = 0$$

Mode-to-mode coupling

↓

backreaction + rescattering

- Non-perturbative process : numerical tools (Lattice)

IR cutoff $k_{min} \sim (2\pi)/L$ UV cutoff $k_{max} \sim N(2\pi)/L$

Parametric resonance → turbulence → thermalization

(Non thermal spectrum) $\langle \chi^2 \rangle \propto t^{-2n}$

Khlebnikov & Tkachev '96; Prokopec & Roos '97;
Felder et al. '01; Micha & Tkachev '02

- Einstein Equations:
  non-linear coupled system matter - metric

Preheating of field fluctuations may lead to:

- Amplification of super-Hubble perturbations (primordial spectrum)

  Tsujikawa & Bassett '02

- Non-gaussianity

  Enqvist et al. '05 ; Barnaby & Cline '06
  Chambers & Rajantie '08

- **Stochastic background of gravitational waves**

  Easther, Giblin & Lim PRL99 '07; PRD77 '08
  García-Bellido & Figueroa PRL98 '07
  García-Bellido, Figueroa & Sastre PRD77 '08
  Dufaux et al. PRD76 '07; JCAP 03 '09
  MBG, Macías-Pérez,Santos PRL105 '10

Previous studies $ds^2 = dt^2 - a(t)^2(\delta_{ij}(1+2\Psi)+h_{ij})dx^i dx^j$

$\ddot{h}_{ij} + 3H\dot{h}_{ij} - \nabla^2 h_{ij}/a^2 = S_{ij}^{TT}/a^2$ transverse & traceless source term

$S_{ij}^{(\Psi)} \ll S_{ij}^{M}$ Only matter (fields)

- Fields evolve in a background metric

- Easther & Giblin & Lim

source $S_{ij}^{M}(t,x) \sim 16\pi G \nabla_i \chi \nabla_j \chi$ FT ➡ $S_{ij}^{TT}(t,k) = \Lambda_{ij,lm}(k) S_{ij}(t,k)$

Tensors evolve in momentum space

- Dufaux & Bergman & Felder & Kofman & Uzan

Green function method: $h_{ij}(t,k) = 16\pi G \int G(t,t') S_{ij}^{TT}(t',k)/k\, dt'$

- García-Bellido & Figueroa & Sastre

Tensors evolve in a lattice with S_{ij} $h_{ij}^{TT}(t,k) = \Lambda_{ij,lm}(k) h_{ij}(t,k)$

Fields + metric perturbations in a lattice

Einstein eq.: $R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = T_{\mu\nu}/m_P^2$ Matter fields $\Phi(x,t)$

- BSSN formalism + synchronous gauge

$$ds^2 = dt^2 - e^{4\beta(t,x)} \tilde{\gamma}_{ij} dx^i dx^j \qquad \mathrm{Det}(\tilde{\gamma}_{ij}) = 1$$

$\langle e^{2\beta(t,x)} \rangle$ ~ a(t) scale factor $\langle 2\dot{\beta}(t,x) \rangle$ ~ H(t) Hubble rate

$\tilde{\gamma}_{ij}(t,x) \sim \delta_{ij} + h_{ij}(t,x)$ ~ tensors

- EOM: $\ddot{\beta} + 2\dot{\beta}^2 = -(\rho + 3P)/(3m_P^2) - \dot{\tilde{\gamma}}_{ij} \dot{\tilde{\gamma}}^{ij}/24$

$$\ddot{\tilde{\gamma}}_{ij} + 6\dot{\beta}\dot{\tilde{\gamma}}_{ij} = 2e^{-4\beta}(M_{ij}^{TF} - R_{ij}^{TF}) + \dot{\tilde{\gamma}}_{il}\dot{\tilde{\gamma}}^{l}_{j}$$ **Traceless**

fields **metric**

➡ Fields ➡ scalar β : $M_{ij}^{TF} \simeq R_{ij}^{TF}$

$$\begin{cases} M_{ij}^{TF} \simeq m_P^{-2}(\partial_i \Phi \partial_j \Phi)^{TF} \\ R_{ij}^{TF} \simeq (-4\partial_i\beta\partial_j\beta + 2\partial_i\partial_j\beta)^{TF} \end{cases}$$

➡ $\dot{\tilde{\gamma}}_{ij}^{TT}(t,k) = \Lambda_{ij,lm}(k)\dot{\tilde{\gamma}}_{ij}(t,k)$ **Transverse & Traceless**

(MBG, Macías-Pérez,Santos PRL105 '10)

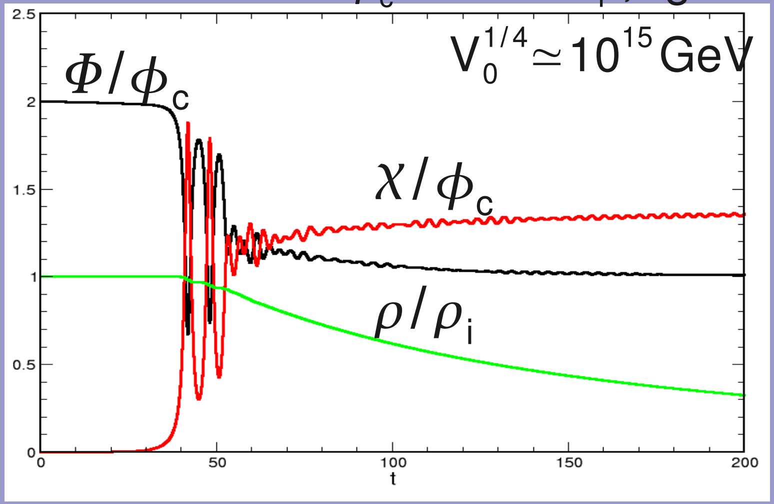
Fields + metric perturbations: Hybrid model

$$V = V_0 + \frac{g^2}{4}\chi^4 + g^2(\Phi^2 - \phi_c^2)\chi^2 + \frac{1}{2}m_\phi^2\Phi^2$$

$$\Phi < \phi_c: \quad m_\chi^2(\Phi) < 0$$

Tachyonic preheating: very efficient particle production mechanism $\delta\phi_k \simeq \delta\chi_k$

$\phi_c = 0.005 m_P$, $g = 0.01$,



- Initial conditions:

  No tensors at t=0,
  local scale factor and Hubble rate from constraint equations
  Fields: slow-roll initial background values + vacuum fluct.

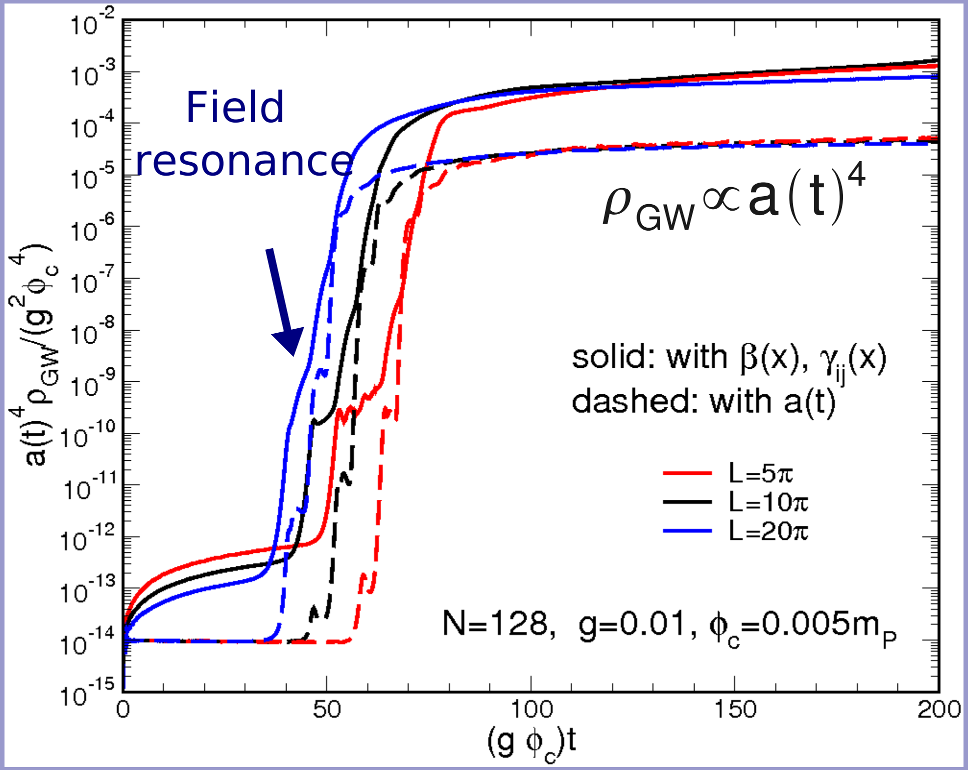
- GW energy density: $\rho_{GW} = m_P^2 \langle \dot{\gamma}_{ij}^{TT} \dot{\gamma}^{ijTT} \rangle / 4$

Power spectrum: $$\frac{d\Omega_{GW}}{d\ln k} = \frac{1}{\rho_c}\frac{d\rho_{GW}}{d\ln k} = \frac{\pi k^3}{3H^2L^2}|\dot{\gamma}_{ij}^{TT}(k)\dot{\gamma}^{ijTT}(k)|$$

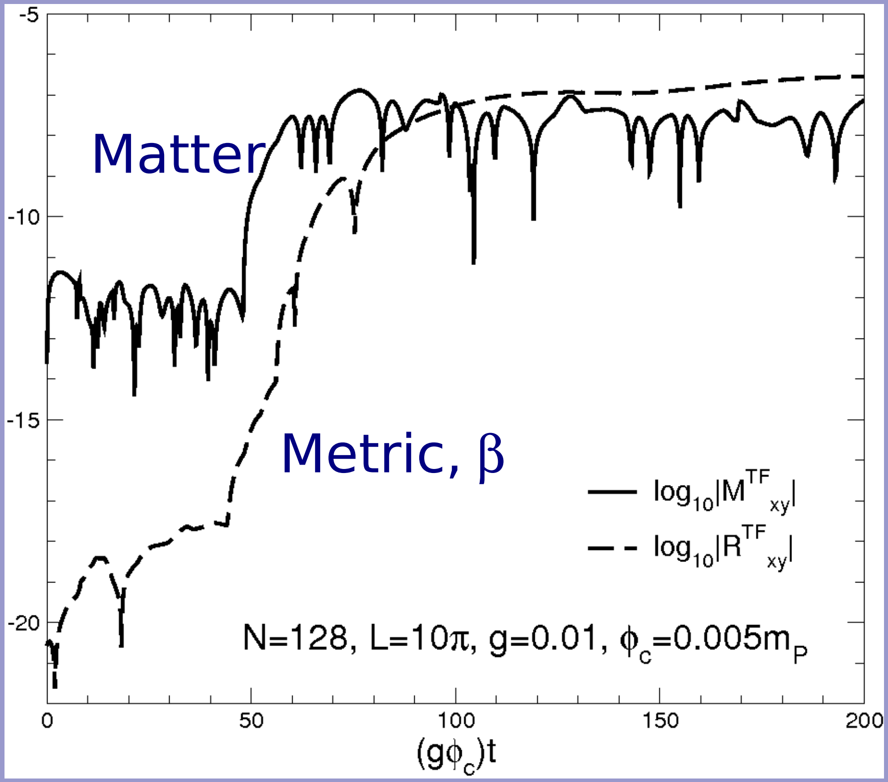
Peak at k_{res} $\sim g\,\Phi_c$ $\quad$ $\rho_{GW} \sim$ $H_{res}/m_P \sim g\,\Phi_c^2/m_P^2$

Hybrid model

GW energy density



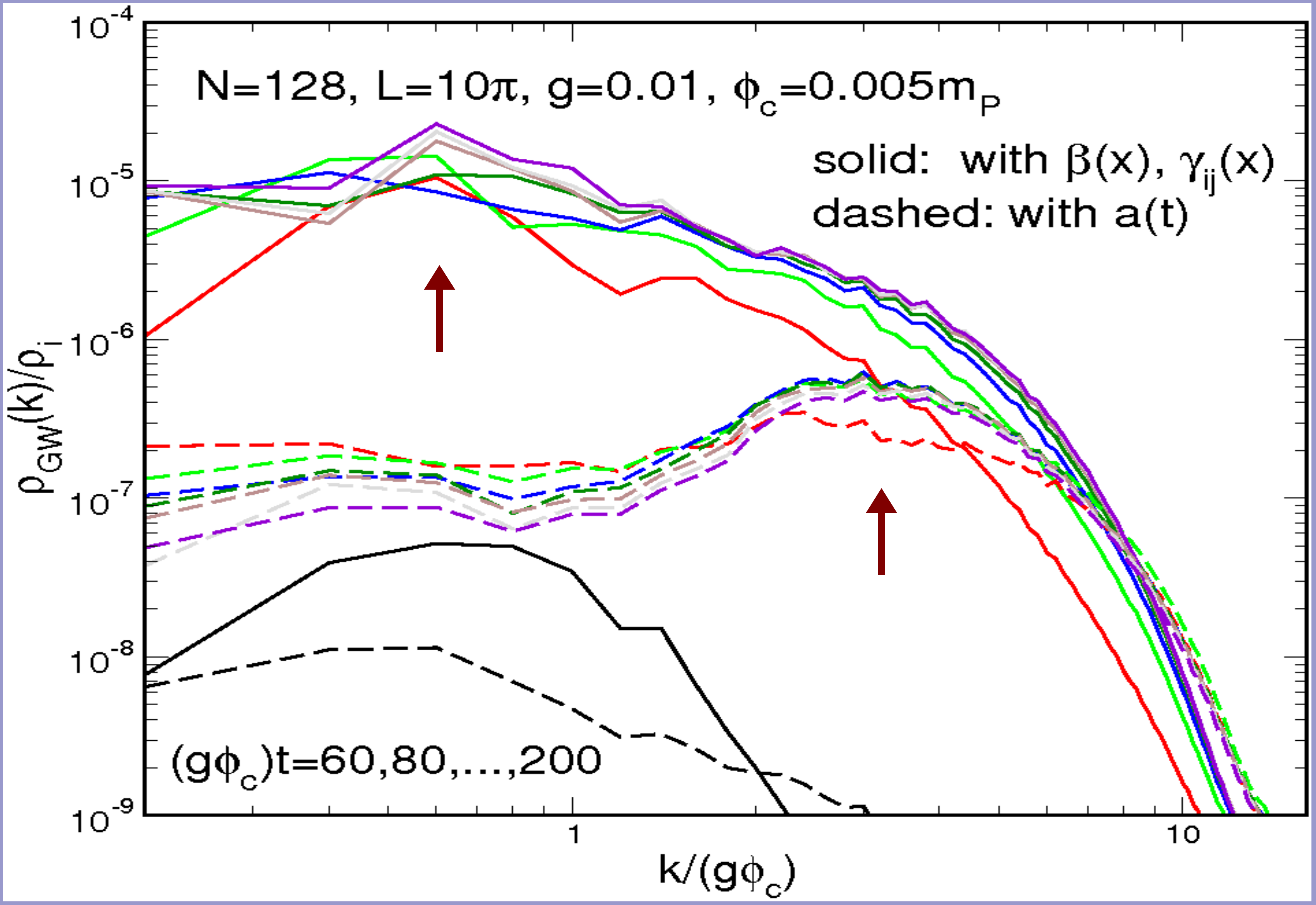
Source terms



- larger ρ_{GW} (~ an order of magnitude) when including metric perturbations

(Cluster UGRGrid, Granada & BlueGeneP, IDRIS, France)

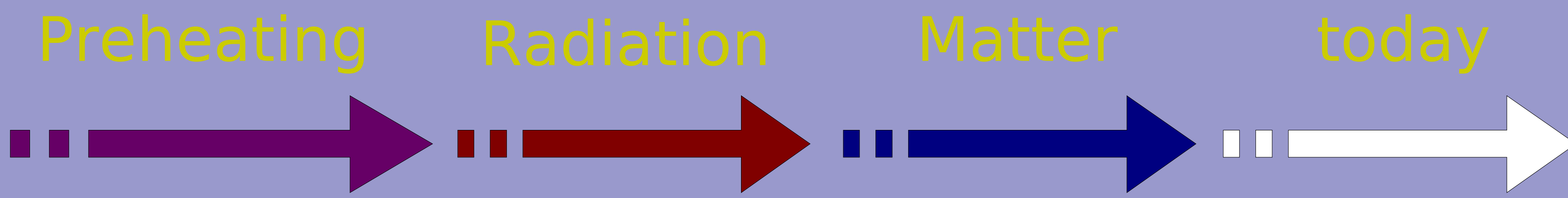
Hybrid model : power spectrum



- $k_{res} \sim (g\,\Phi_c)$, shifted towards smaller values
- larger lattice needed to improve infrared resolution with a reliable ultraviolet cutoff

($k_{min}=0.4g\,\Phi_c$)

- From preheating to today values



No entropy production

$$g_S(T_R)\,a^3(t_R)\,T_R^3 = g_S(T_0)\,a^3(t_0)\,T_0^3$$

Frequency:

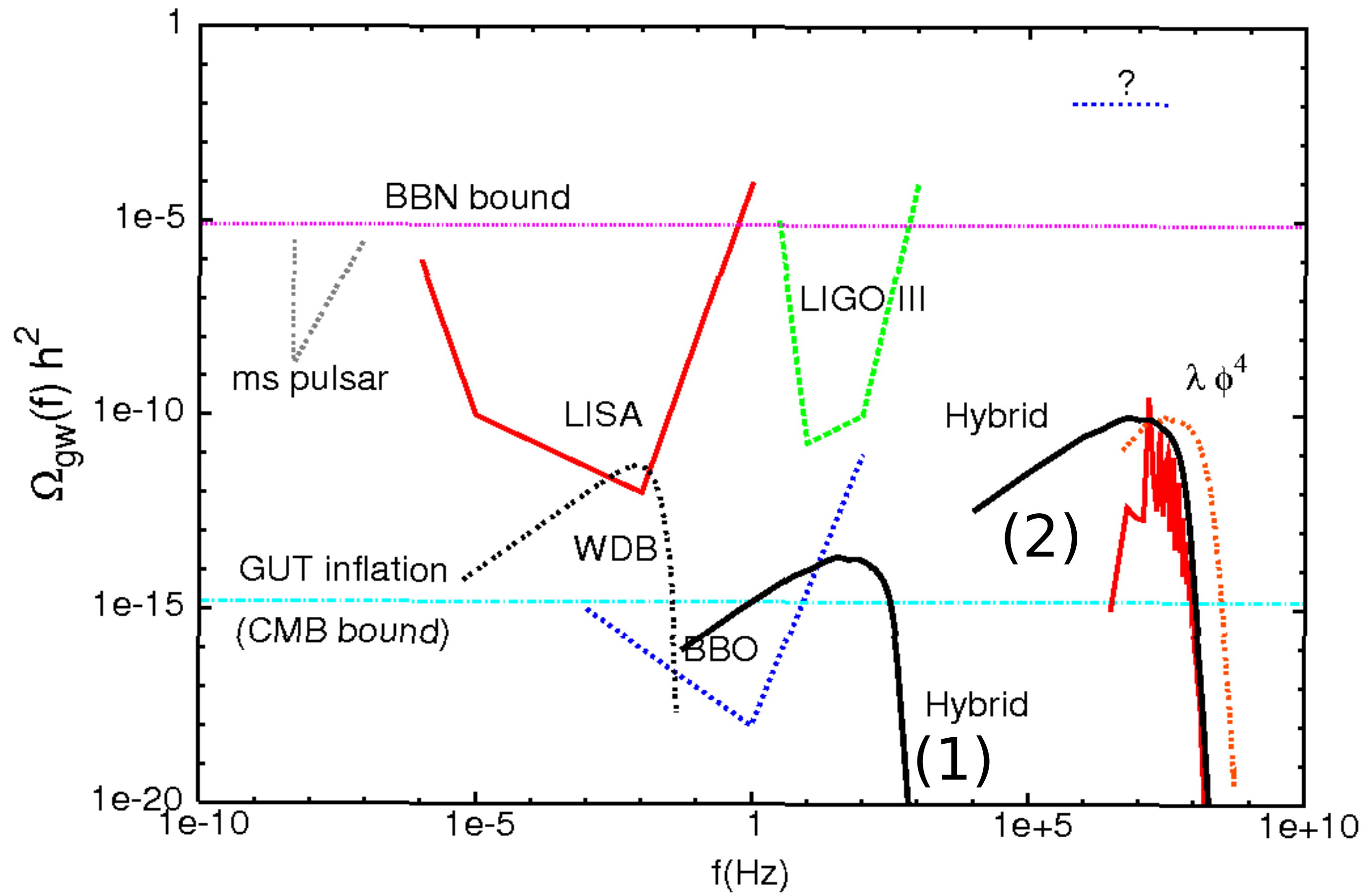
$$f_0 = \frac{k_0}{2\pi} \simeq \frac{k_{res}}{2\pi}\frac{a_{res}}{a_0} \simeq 6.4\times10^{10}\sqrt{g}\,\frac{k_{res}}{\rho_{res}^{1/4}}\,\mathrm{Hz} \sim 10^{10}\sqrt{g}\,\mathrm{Hz}$$

Power spectrum:

$$\Omega_{GW}h^2 = \Omega_{GW}^{(res)}h^2\left(\frac{a_{res}}{a_0}\right)^4 \simeq 5.5\times10^{-9}\left(\frac{\phi_c/m_P}{0.005}\right)^2$$

Low scale hybrid inflation (very weak coupling $g < 10^{-14}$) could lead to a stochastic background of GW within the reach of GW detectors $f_{detec} < 10^3$ Hz

Sensitivity of planned GW interferometers detectors



(1) $\Phi_0 \sim 10^{-5}\, m_P$, $g \sim 10^{-14}$ (2) $\Phi_0 \sim 10^{-3}\, m_P$, $g \sim 10^{-2}$

Very weak coupling $g << 1$ ➡ tensor-to-scalar $r << 1$

García-Bellido, Figueroa, Sastre 0707.0839

Summary

- Preheating: parametric amplification of field + metric fluctuations during oscillations after inflation

$$\ddot{\Phi}+6\dot{\beta}\dot{\Phi}-e^{-6\beta}\nabla^2\Phi+V_\phi-2e^{-4\beta}\nabla^i\beta\nabla_i\Phi=0$$

$$\ddot{\beta}+2\dot{\beta}^2\simeq-(\rho+3P)/(3m_P^2)$$

$$\ddot{h}_{ij}+3H\dot{h}_{ij}-e^{-2\beta}\nabla^2 h_{ij}\simeq S_{ij}^{\Phi}/m_P^2+S_{ij}^{\beta}$$

$$|\beta_k|^2\sim(\phi_c/m_P)^2|\phi_k|^2$$

Scalar metric perturbations follow the same resonance pattern than the fields

Tensors sourced by 2nd order matter and metric perturbations:

$$S_{ij}^{\beta}\sim S_{ij}^{\Phi}/m_P^2$$

- Coupled system fields + (scalar, tensor) metric fluct:

  ~ 1 order of magnitude enhancement for ρ_{GW}

  ~ frequency shifted to smaller values

- Next step:

  improve infrared and ultraviolet limits

  Vectors? Magnetic fields?

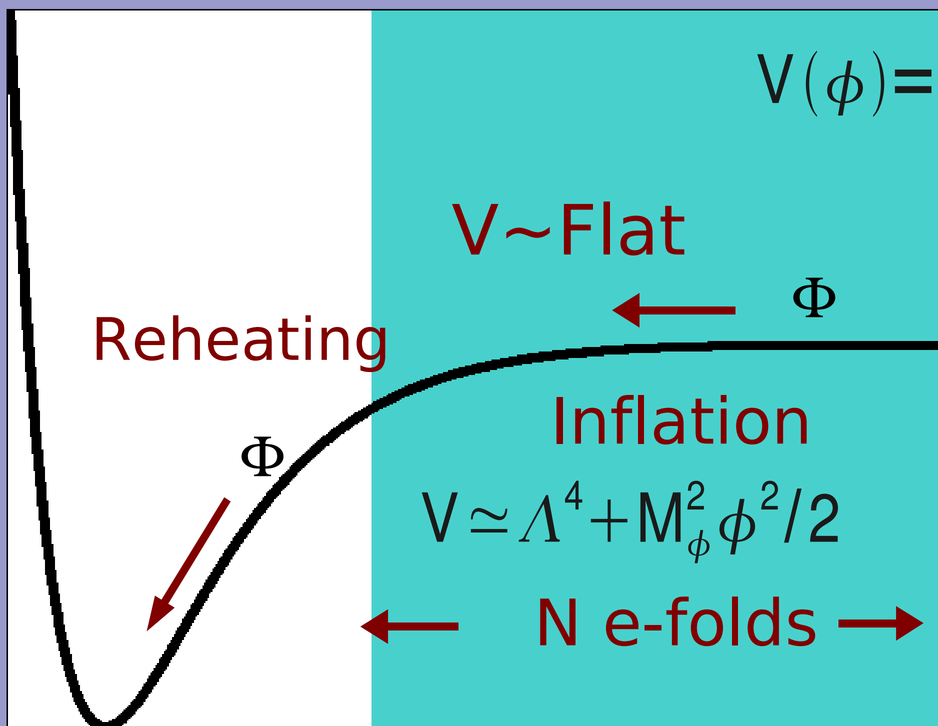
  Non-gaussianity?

Hybrid inflation (2 fields models)

Inflation ➡ phase transition ➡ reheating

False vacuum

Global minimum



$$V(\phi)=\Lambda^4+\frac{M_\phi^2}{2}\phi^2+\frac{\lambda}{4}\chi^4+\frac{1}{2}\left(g^2\phi^2-\sqrt{\lambda}\,\Lambda^2\right)\chi^2$$

Inflation: above the critical value

The second field vanishes
Potential ~ vacuum energy

Global minimum: below critical

The second field gets a vev
The potential vanishes

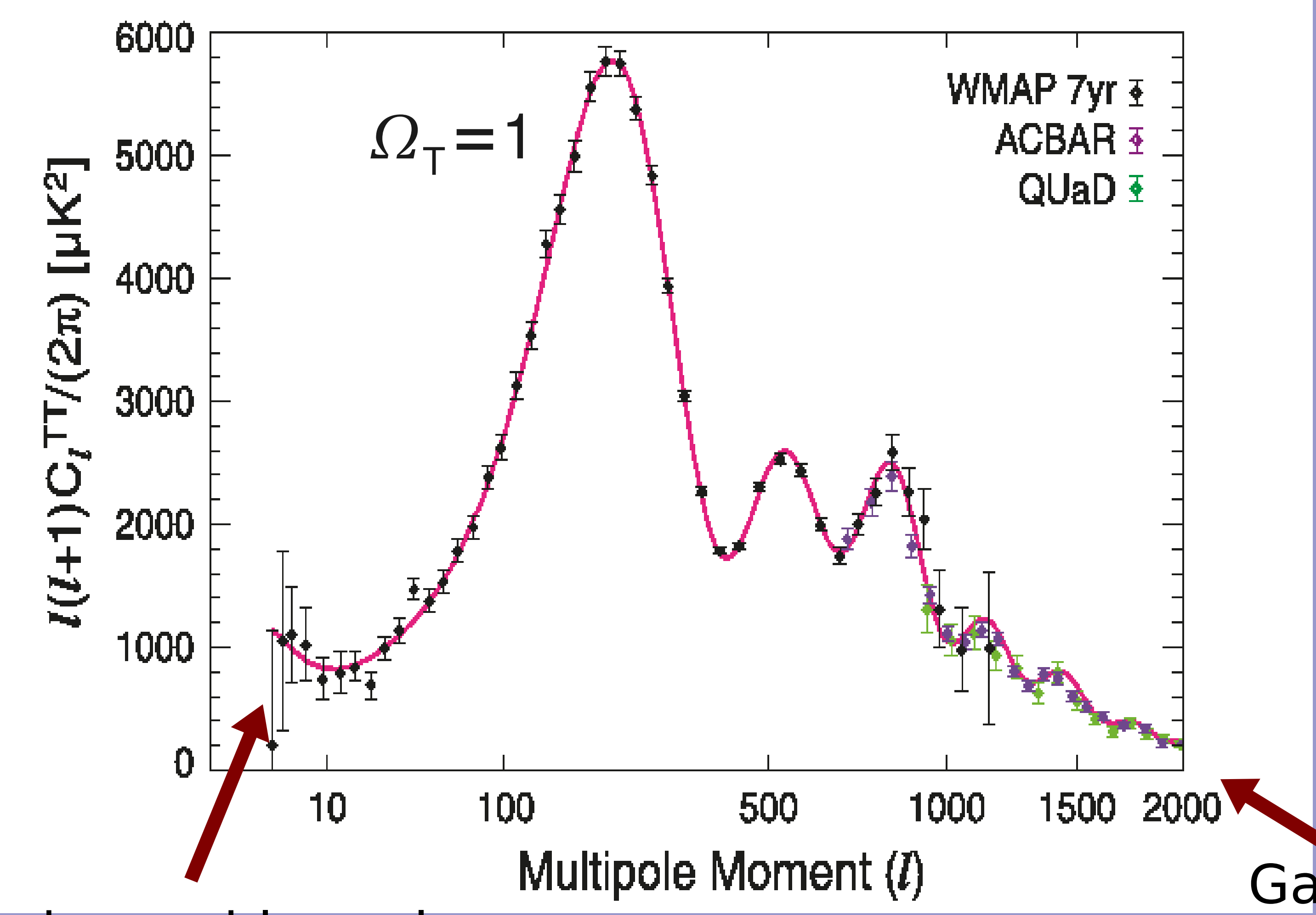
Small (below Planck) field values

(No tensor: r << 1)

Linde, PLB’90
Copeland et al, PRD’94

Cosmic Microwave Background Radiation

Wilkinson Microwave Anisotropy Probe



T Anisotropy Power

COBE (1992)
Boomerang
Maxima
DASI
CBI
VSA
•
•
•
PLANCK
(14/05/09)

Largest observable scale
O(3000 Mpc)

Galaxy scale
O(Mpc)

astro-ph/1001.xxxx