Particles, Strings and Gauge Theories

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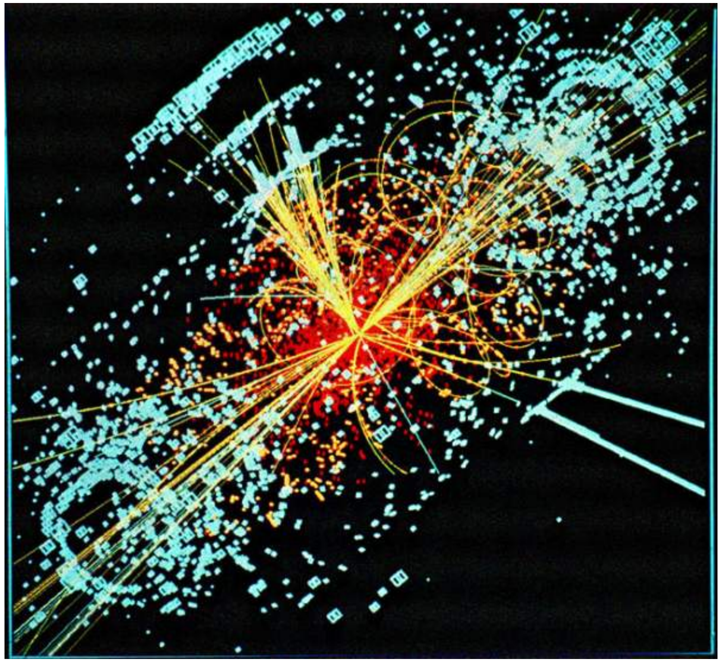
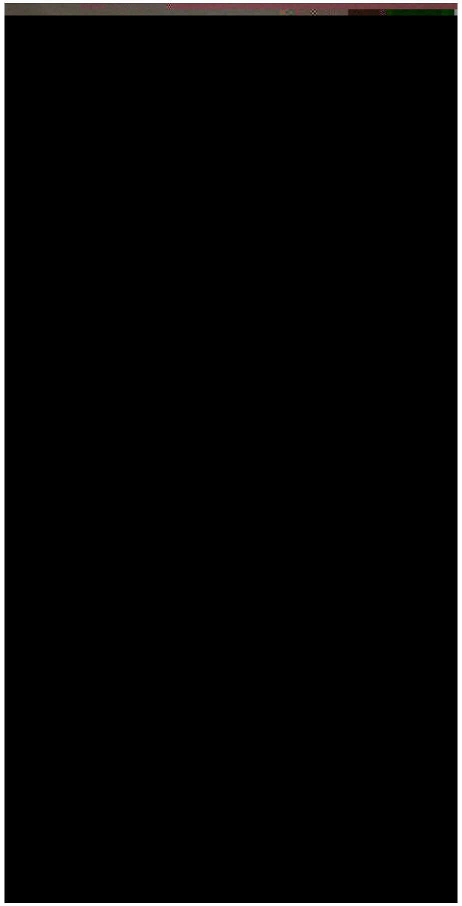
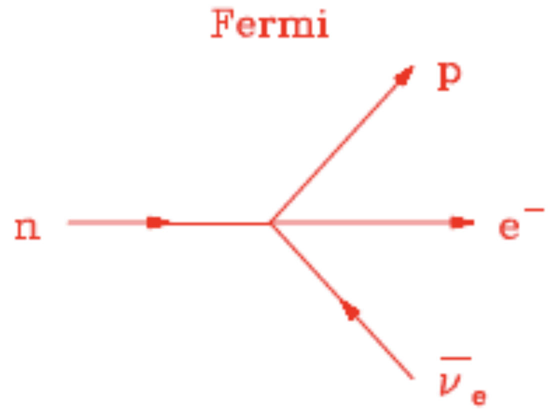




What have we learned from the Standard Model of Particle Physics ?

Fermi's model for beta-decay



$$h = g\psi_P{}^*\psi_N \cdot \psi_\epsilon{}^*\psi_\nu.$$

Good tree-level predictions

Problems with radiative corrections

Effective Lagrangian Lore

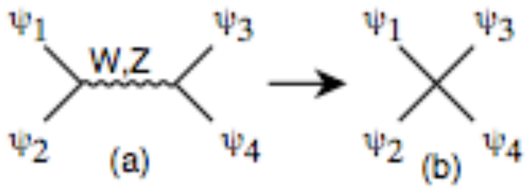


Fig. 1. (a) Tree level W and Z exchange between four fermions. (b) The effective vertex in the low energy effective theory (Fermi interaction).

Integrating out W

$$g\bar{\psi}\gamma^\mu A_\mu \psi \rightarrow \frac{g^2}{M^2}\left(\bar{\psi}\psi\right)^2$$

$$M \sim 90\,\frac{GeV}{c^2}$$

Effective theory (Fermi)

$$O\left(\frac{E^2}{M^2}\right)$$

Weinberg's Folk Theorem

"If one writes down the most general possible Lagrangian, including all terms consistent with the assumed symmetry principles, and then calculates matrix elements with this Lagrangian to any given order in perturbation theory, the result will simply be the most general possible S-matrix consistent with perturbative unitarity, analiticity, cluster decomposition and the assumed symmetry principles"

Naive Dimensional analysis

$\hbar = c = 1$

$$[E = mc^2] = [M] \equiv 1$$

All dimensions are expressed in powers of M

$$[Et] = \hbar = 0 \qquad [x = ct] = [t] = \frac{1}{E} \qquad [\partial_\mu] \equiv [\frac{\partial}{\partial x^\mu}] = 1$$

$$[\frac{S}{\hbar}] = 0$$ (Feynman) $$S = \int d^4 x L \qquad \Rightarrow [L] = 4$$

Kinetic energy determines dimensions of fields:

$$L_K = \frac{1}{2}\partial_\mu \phi \partial^\mu \phi \qquad [\phi] = [m] = 1$$

Lagrangian as a sum of operators

$$L = \sum_{n=0}^{\infty} \frac{g(\Lambda)^n}{\Lambda^n} \mathcal{O}^{(n+4)}$$

Dimensional analysis with quantum corrections

$n \leq 0$ **Relevant (masses)** $\frac{m^2}{2}\phi^2$ (Yukawa potential) $O\left(\frac{\Lambda^2}{E^2}\right)$

Increase in the infrared (E<<M)

$n \geq 0$ **Irrelevant (NR)** $\frac{g}{M}\phi^5$ $O\left(\frac{E}{\Lambda}\right)$

(Decrease in the IR)

$n = 0$ **Marginal** $\frac{\lambda}{24}\phi^4$ $O(1)$

The dependence on the cutoff is such that physics is independent of it

$$\Lambda \frac{\partial}{\partial \Lambda} c_n(\Lambda) = \beta_n(c_n(\Lambda))$$

Ultraviolet critical surface

$$\Lambda \to \infty \quad \Rightarrow \quad c \to c^*$$

Each critical point defines an UNIVERSALITY CLASS

Effective theories useful for $\frac{E}{\Lambda} \leq 1$

Once a symmetry is specified, quantum corrections enforce all terms in the effective lagrangian compatible with the symmetries

Naturalness: $\Longleftrightarrow \quad \forall i, c_i \sim 1$

$$ds^2 = g_{\mu\nu} dx^\mu dx^\nu \quad [g_{\mu\nu}] = 0 \quad [\Gamma] = 1 \quad R^\mu_{\nu\rho\sigma} \sim \partial\Gamma + \dots \; [R] = 2$$

$$L_{eff} = c_0 \Lambda^4 \sqrt{|g|} + c_1 \Lambda^2 R \sqrt{|g|} + c_2 R^2 + \frac{1}{2} g^{\alpha\beta} \nabla_\alpha \phi \nabla_\beta \phi \sqrt{|g|} + \\ + c_3 \frac{1}{\Lambda^2} R^{\alpha\beta} \nabla_\alpha \phi \nabla_\beta \phi \sqrt{|g|} + c_4 \frac{1}{\Lambda^2} R^3 \sqrt{|g|} + c_5 \phi^4 \sqrt{|g|} \dots$$

The scale is fixed by the value of the coefficient of the Einstein-Hilbert term (i.e. Planck's mass)

$$c_1 \Lambda^2 = -\frac{c^3}{16\pi G} \equiv -2M_p^2 \qquad M_p \sim 10^{19} GeV$$

Irrelevant terms make a contribution $O\left(\left(\frac{E}{M_p}\right)^n\right)$

When E=Mp all terms in the expansion are of the same order.

This strongly suggests that it is impossible to understand quantum gravity in isolation from all other interactions

(No decoupling limit in sight)

Steps in the road to unification

ELECTROMAGNETISM (MAXWELL)

$$\nabla \cdot \mathbf{E} = 0$$
$$\nabla \cdot \mathbf{B} = 0$$
$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$
$$\nabla \times \mathbf{B} = \mu_0 \varepsilon_0 \frac{\partial \mathbf{E}}{\partial t}.$$

Symmetry E versus -B ?

Dirac quantization condition

$$\frac{q_e q_m}{2\pi\epsilon_0 \hbar c^2} \in \mathbb{Z}$$

't Hooft-Polyakov monopole (Nonsingular)

Montonen-Olive

$$\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{g^2}.$$

$$\tau \mapsto \frac{-1}{n_G \tau}$$

ELECTROWEAK UNIFICATION (GWS)

Dark matter & Dark energy

Landau pole

Neutrino Masses

Naturalness

New Physics BSM

EXTRA DIMENSIONS (KALUZA-KLEIN)

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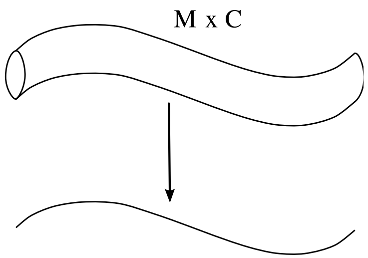


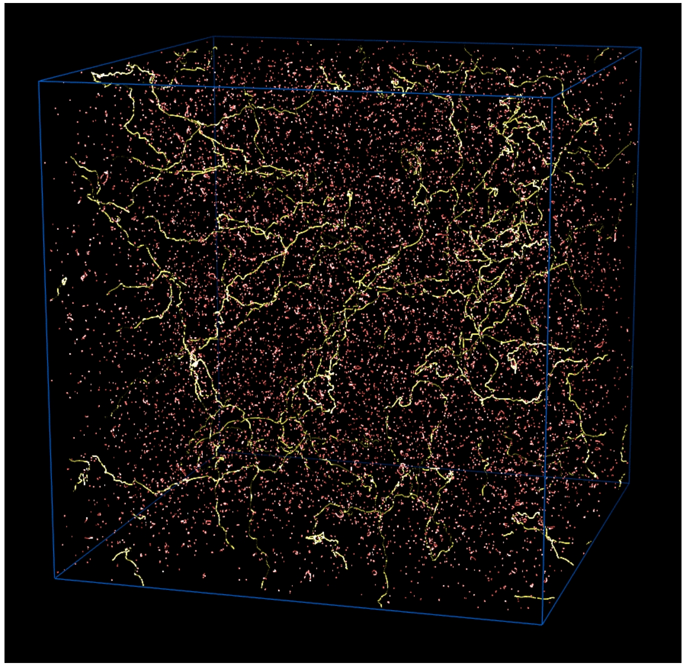
$$\hat{g}_{MN} = \begin{pmatrix} g_{\mu\nu} + \Phi A_\mu A_\nu & \Phi A_\mu \\ \Phi A_\nu & \Phi \end{pmatrix}$$

Dilaton enters the scene

GRAND UNIFIED THEORIES (GEORGI-GLASHOW)

Cosmic (super)strings (Nielsen-Olesen)

$$\pi_1\left(G/H\right)$$



(Allen-Shepard)

(cf. Achucarro)

No proton decay observed

SUPERSYMMETRY (WESS-ZUMINO)

A new spacetime symmetry

CC=0

Neutrinos are not (pseudo)goldstinos (VA)

Naturalness of SM

SUPERGRAVITY (FFPVN) (Local susy)

SUPERSTRINGS (GREEN-SCHWARZ)

Small digression: Quantum fields on shell=Feynman integrals over particle trajectories

$$\int \frac{dT}{T} N \int \mathcal{D}x(\tau) \quad e^{-\int_0^T d\tau \left(\frac{\dot{x}^2}{2e} + e\frac{m^2}{2}\right)} = -log\, det\, \left(-\Box + m^2\right)$$

QFT Off Shell: Vacuum Structure, SSB

STRINGS ON SHELL

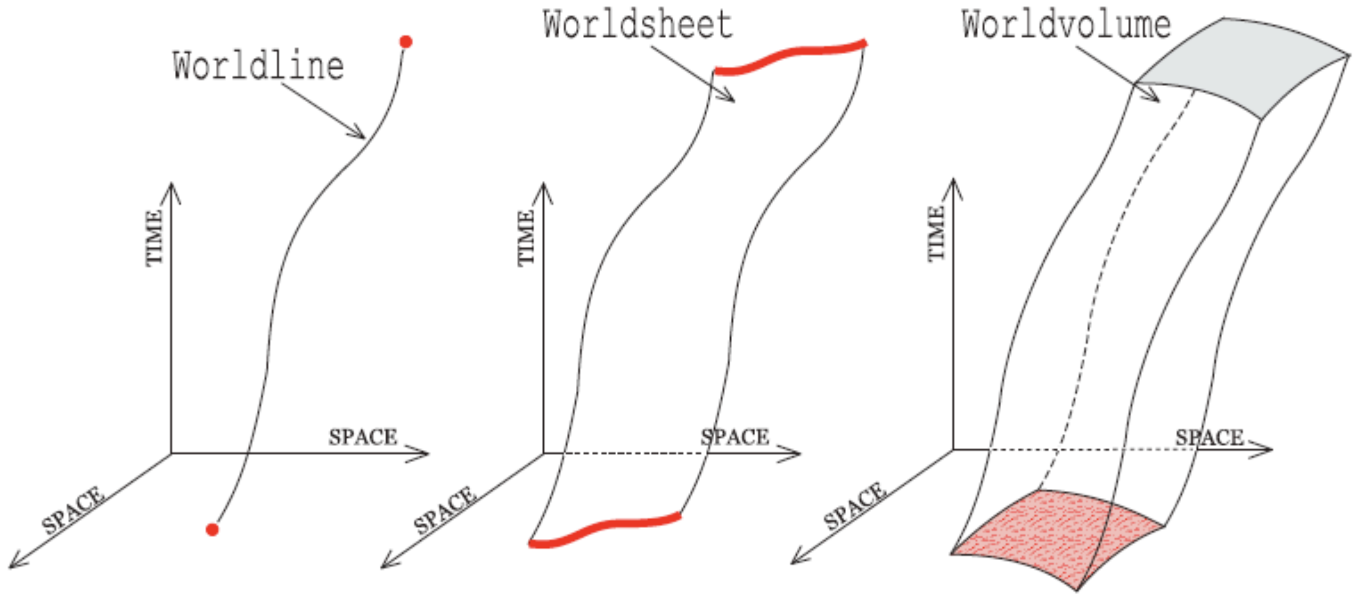


Figure 14: Particles, strings and membranes.

Strings do have a contrived history

Strong interactions (Veneziano)

Large N Strings (' t Hooft)

Fundamental Strings (Scherk-Schwarz)

Cosmic Strings (SSB)

World-sheet approach (CFT)

Embedding in spacetime $X^{\mu}(\xi_{\alpha})\ (\mu = 0, 1, .., D-1)$

Action

$$S = -\frac{T}{2}\int d^{\mathrm{p}+1}\xi\,\sqrt{-\det h}\,\{h^{\alpha\beta}\partial_{\alpha}X^{\mu}\partial_{\beta}X$$
$$-(\mathrm{p}-1)\}\,,$$

Mode expansion

$$X^{\mu} = X_0^{\mu} + P^{\mu}\tau \qquad (1.1$$
$$+\frac{i}{\sqrt{2}}\sum_{n\neq 0}\frac{1}{n}\{a_n^{\mu}e^{-in(\tau+\sigma)} + \tilde{a}_n^{\mu}e^{-in(\tau-\sigma)}\}$$

GGRT: D=26

Open spectrum

$$
\begin{array}{llll}
N=0 & |p> & -\frac{1}{4}p^2 = -1 & \text{tachyon} \\
N=1 & a^{\mu}_{-1}|p> & -\frac{1}{4}p^2 = 0 & \text{massless vector} \\
N=2 & \left\{ \begin{array}{c} a^{\mu}_{-1}a^{\nu}_{-1}|p> \\ a^{\mu}_{-2}|p> \end{array} \right. & -\frac{1}{4}p^2 = 1 & \text{massive spin } 2\,,
\end{array}
\qquad (1.30)
$$

NSR&GSO

Susy: D=10

Closed spectrum

$$a^{\mu}_{-1}\tilde{a}^{\nu}_{-1}|p>$$

Graviton

Dilaton

Kalb-Ramond

Strings do not have UV divergences

Modular invariance: UV ⟷ IR

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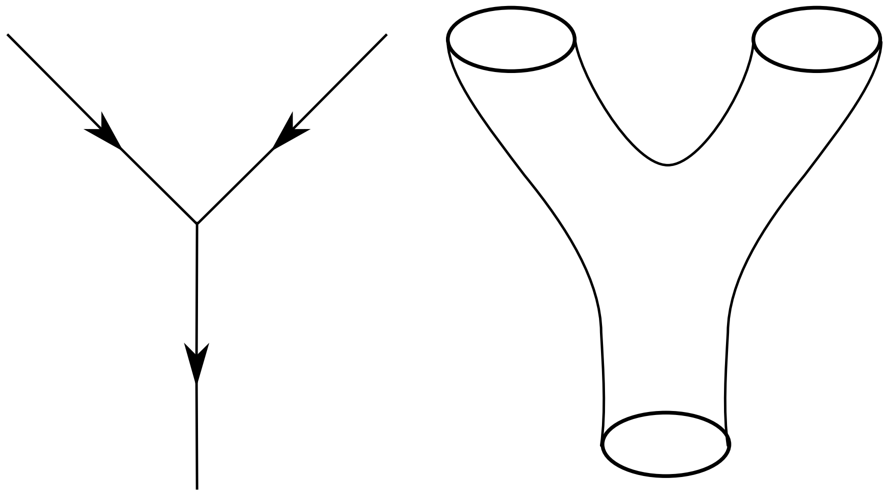


Consistency relies on conformal invariance & spacetime SUSY

T-DUALITY

(Winding states)

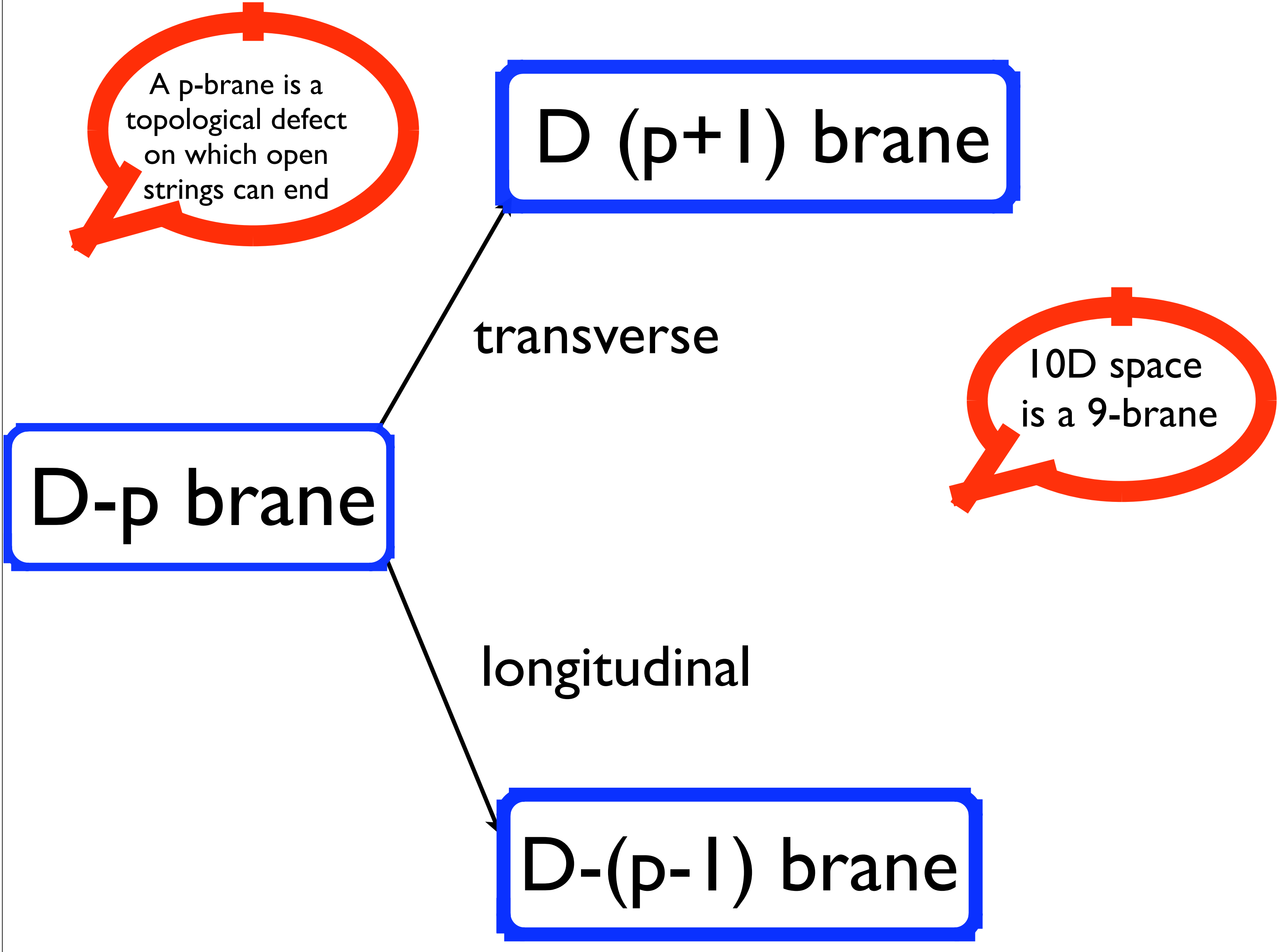
$$X = X_0 + p\tau + w\sigma + \frac{i}{\sqrt{2}} \sum_{n \neq 0} \frac{1}{n} \{a_n e^{-in(\tau+\sigma)} + \tilde{a}_n e^{-in(\tau-\sigma)}\} \quad (1.35)$$

$$p_{L,R} = \frac{m}{R} \pm nR \,,$$

Momentum versus winding

$$R \to \frac{1}{R} \,, \; m \leftrightarrow n \quad \text{or} \quad X_L \to X_L \,, \; X_R \to -X_R \,. \quad (1.38) \qquad \lambda \to \frac{\lambda}{R} \,.$$

This a stringy symmetry, valid to all orders in perturbation theory


A p-brane is a topological defect on which open strings can end
D (p+1) brane
transverse
10D space is a 9-brane
D-p brane
longitudinal
D-(p-1) brane

Conformal invariance=Einstein's equations

$$S = -\frac{1}{4\pi} \int d^2\xi \{ G_{\mu\nu}(X)\partial_\alpha X^\mu \partial^\alpha X^\nu \quad (1.72$$
$$+ B_{\mu\nu}(X)\epsilon^{\alpha\beta}\partial_\alpha X^\mu \partial_\beta X^\nu$$
$$- \phi(X)\mathcal{R}^{(2)} + A^a_\mu(X) J^\mu_a + \ldots \},$$

Graviton

Kalb-Ramond

Dilaton

Universal backgrounds

$$S_n \equiv \int d(vol)_n \frac{1}{2\kappa_n^2} \, e^{-2\phi} \left[R - \frac{n-26}{3l_s^2} - \frac{1}{12} H_{abc}H^{abc} + 4(\nabla\phi)^2 \right]$$

Action principle for consistency of string propagation

Dilaton= String coupling constant

Dilaton shift $\phi \rightarrow \phi + c$

$$\frac{1}{4\pi}\int d^2\xi \mathcal{R}^{(2)} = 2(g-1)\,.$$ Euler

String loop expansion

$$e^{-S} \rightarrow e^{2c(g-1)} e^{-S}\,,$$

Contribution of all genera

It follows that the dilaton shift can be absorbed in a rescaling of the string coupling $\lambda \rightarrow e^c\lambda$. The 10d effective action can therefore by expanded in powers of e^{ϕ} corresponding to the perturbative topological string expansion:

Sigma model expansion= powers of alpha prime

S-Duality

Generalization of electric-magnetic

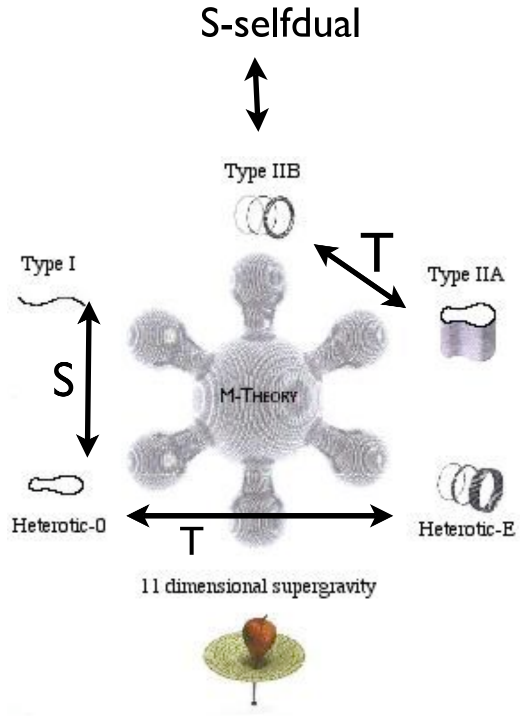
$$D_p \longrightarrow A_{p+1} \longrightarrow F_{p+2} \xrightarrow{Poincare} F^*_{n-p-2} \longleftarrow A^*_{n-p-3} \longleftarrow \tilde{D}_{n-p-4}$$

Strong-weak duality

$$g_s \longrightarrow \frac{1}{g_s}$$

$$\phi \longrightarrow -\phi$$

Conjectured web of dualities



$Sugra_1^{11}$ No coupling constants

10-D dilaton given by the radius of the eleventh dimension $R_{11} = g_s l_s$

AdS/CFT

World-sheet versus spacetime description of D-3 branes

4-D SYM^4_4

SU(N)

Maldacena ⟷

D=10

IIB $AdS_5 \times S_5$

$$g_{YM}^2 = 4\pi g_s$$

$$\frac{l}{l_s} = (4\pi g_s N_c)^{\frac{1}{4}}$$

$$\lambda \equiv g_{YM}^2 N = 4\pi g_s N$$

$$G_{10} \sim g_s^2 l_s^8 \sim \frac{l^8}{N^2}$$

$$l \sim N^{\frac{1}{4}} G^{\frac{1}{8}}$$

't Hooft coupling large

Large radius and small curvature (SUGRA approximation)

Down to earth applications?

Strongly coupled plasmas

The holographic picture of a strongly coupled plasma is the physics of a generic dynamical black hole horizon, after a certain mathematical "translation" is made. It gives the natural starting point to discuss near-perfect fluids with very strong coupling.

The main difficulty is that systematic errors are difficult (perhaps impossible?) to estimate.

Condensed Matter

One dominant paradigm in many-body theory is the identification of critical points described by conformal field theories and the study of perturbations thereof.
AdS/CFT provides a parametrization of a very large class of quantum critical systems with very strong coupling.
It comes with a new type of "mean field method" that could succeed where others failed.

Its main advantage as well as its main drawback is its genericity

The opportunity: Maybe some exotic systems (high Tc superconductors?) could fall in one of the new universality classes described by AdS/CFT.

The long term dream: maybe a quantum-gravitational black hole could be "engineered" using a complicated Condensed Matter system on a table top.

What have we learned from strings?

There is much to learn about gauge theories

Gravity versus renormalization group

There is a smooth description of some singularities

Spacetime is not a good short distance description (Emergent?)

The sigma model description does not capture the conjectured web of dualities

Main problem: Susy

There are virtually no general predictions below the scale of Susy breaking

Related problem: Compactification

Below the susy breaking/compactification scale: effective field theories

Susy and de Sitter

No global timelike killing (Horizon)

$\therefore$ It is not possible to implement $Q^2 = H$

Euclidean continuation: $e^{2\pi K} = 1$

Thermal effects observer dependent

We do not understand the vacuum, even in the SM

Strong Interactions	$\lvert\bar{\theta}\rvert < 3 \times 10^{-10}.$
Electroweak	(Neutrino) masses
Gravitational	Cosmological Constant
Strings	Landscape of vacua

``If we really live in a multiverse, Physics will have been reduced to an environmental science like Botany.''

Gell-Mann

Conclusions: Strings are fascinating constructs poorly undestood as yet.
They suggest a complicated nongeometrical structure at small distances

Spinoffs

- A new way of looking to Feynman diagrams (Bern et al, Arkani-Hamed et MHV)
- Mirror manifolds (Candelas et al: countings of rational curves)
- Susy Field Theory dualities (Seiberg-Witten)
- Conformal Field Theory (Belavin Polyakov Zamolodchikov)
- Gravity=Gauge.Gauge (Kawai Llewellen Tye)
- BH entropy=3 tangle (Duff et al)

Strings perhaps are a piece of XXI century physics that has been unveiled too soon...

Strings perhaps are a piece of XXI physics which has been unveiled too son

or else...

the whole approach is misguided and we are left with the spinoffs only.

Experiment will tell

"Sorry. I know you've got charm
but I'm really into branes."

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string theorist arrives home one evening. When he goes into his house, his wife tells him that she's hired a private detective who has been following him for the past week and she now knows he's having an affair with another woman.

"But darling…" says the string theorist. "I can explain everything."

Not a very good prediction:

$$\frac{M_p}{M_H} \sim 10^{61}$$

Even

$$\frac{M_p}{M_{DE}} \sim 10^{31}$$

In order to reproduce observations, c0 must be unnaturally small

Caveats

- Effective lagrangians may fail when horizons are present (infinite redshift).
- Reasoning is different when spacetime dimension is bigger than 4 (spontaneous compactification).
- Presence of topological defects (the universe itself?)
- Perhaps a metastable state is ok for cosmological purposes if lifetime long enough.

This is the natural value for a cosmological constant

$$R_{\mu\nu} - \frac{1}{2}(R + 2\lambda)g_{\mu\nu} = \frac{1}{M_p^2}T_{\mu\nu}$$

$$R_{\mu\nu} - \frac{1}{2}R = \frac{1}{M_p^2}\left(T_{\mu\nu} + \lambda M_p^2 g_{\mu\nu}\right)$$

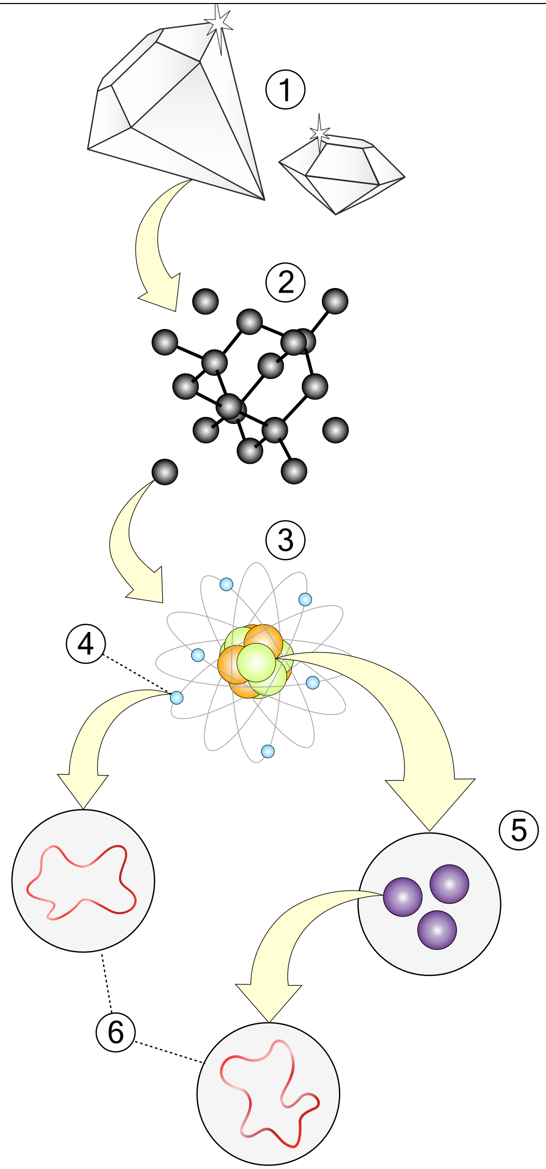
Observations favor

$$\lambda \sim M_H^2 \sim \left(10^{-33}\, eV\right)^2$$

$$M_{DE}^4 \equiv M_H^2 M_p^2 \sim \left(10^{-3}\, eV\right)^4$$

بو الوليد محمد بن احمد بن رشد

Averroes (1126-1198)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