Inflation, Quantum Field Renormalization, and CMB Anisotropies

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Work made in collaboration with: J. Navarro-Salas, G.J. Olmo and L.Parker

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Guth, Hawking, Mukanov, Starobinski, ... '80

Cosmological inflation generates a spectrum of primordial density perturbations that can seed the cosmic structures that we observe today

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Single field slow-roll inflation

$$\phi(t, \vec{x}) = \phi_0(t) + \delta\phi(t, \vec{x}) \longrightarrow \mathcal{R}(t, \vec{x})$$

inflaton perturbations

comoving curvature perturbations
(gauge invariant)

$$\mathcal{R}(t, \vec{x}) = \int d^3k (A_{\vec{k}} \mathcal{R}_k(t) + A^\dagger_{-\vec{k}} \mathcal{R}^*_k(t)) e^{-i\vec{k}\vec{x}}$$

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$t \gg t_k$ Power spectrum $k/a(t_k) = H$

$$\langle 0|\mathcal{R}(t,\vec{x})^2|0\rangle = \int d^3k |\mathcal{R}_k(t)|^2 \equiv \int_0^\infty \frac{dk}{k} \Delta^2_{\mathcal{R}}(k)$$

In momentum space: $\langle 0|\hat{\mathcal{R}}_{\vec{k}}(t)\hat{\mathcal{R}}_{\vec{k}'}(t)|0\rangle = \delta(\vec{k}+\vec{k}')\frac{\Delta^2_{\mathcal{R}}(k)}{4\pi k^3}$

$$\Delta^2_{\mathcal{R}}(k) = \frac{1}{2\epsilon M_P^2}\left(\frac{H}{2\pi}\right)^2$$

$$1 - n_s \equiv \frac{d\ln\Delta^2_{\mathcal{R}}(k)}{d\ln k} = 6\epsilon - 2\eta$$

slow-roll parameters

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Tensor to scalar ratio $\quad r = \dfrac{4\Delta^2_h(k)}{\Delta^2_{\mathcal{R}}(k)} = 16\epsilon$

Consistency relation $\quad r = -8n_t$

Satisfied by ALL single-field slow-roll inflationary models

Reexamining the calculations of the spectrum of perturbations

Momentum space $\langle 0|\hat{\mathcal{R}}_{\vec{k}}(t)\hat{\mathcal{R}}_{\vec{k}'}(t)|0\rangle = \delta(\vec{k}+\vec{k}')\frac{\Delta^2_{\mathcal{R}}(k)}{4\pi k^3}$

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Position space $\langle 0|\mathcal{R}(t,\vec{x})^2|0\rangle = \int d^3k|\mathcal{R}_k(t)|^2 \equiv \int_0^\infty \frac{dk}{k}\Delta^2_{\mathcal{R}}(k)$ UV Divergent!

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Renormalization in curved spacetimes

Renormalization in momentum space removes the divergences by subtracting a set of "counterterms" mode by mode, that is, under the integral symbol

$$\langle 0|\mathcal{R}(t,\vec{x})^2|0\rangle_{\rm ren} = \int d^3k\left(|\mathcal{R}_k(t)|^2 - C_k(t)\right)$$

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All schemes of renormalization in momentum space

Adiabatic regularization (Parker & Fulling 74)

De Witt-Schwinger proper time in momentum space (Bunch & Parker '79)

lead to the same expression for $C_k(t)$

$$\tilde{\Delta}^2_{\mathcal{R}}(k) = \frac{k^3}{4\pi^2}\left(|\mathcal{R}_k(t)|^2 - C_k(t)\right)$$

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We have to specify WHEN the classical perturbations are created

The quantum-to-classical transition takes place a few e-foldings after t_k

If we evaluate the "counterterms" n e-foldings after t_k, but before the end of inflation

$$\tilde{\Delta}^2_{\mathcal{R}}(k) = \frac{1}{2\epsilon M_P^2}\left(\frac{H}{2\pi}\right)^2 (3\epsilon - 2\eta)(2\,n - 3/2)$$

$$\tilde{\Delta}^2_{h}(k) = \frac{2}{M_P^2}\left(\frac{H}{2\pi}\right)^2 \epsilon\,(2\,n - 3/2)$$

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$$\begin{cases} n_s - 1 = -6\epsilon + 2\eta + \dfrac{(12\epsilon^2 - 8\epsilon\eta + \xi^2)}{3\epsilon - \eta} \\ n_t = 2(\epsilon - \eta) \\ r = 16\epsilon\,\dfrac{\epsilon}{3\epsilon - \eta} \end{cases}$$

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New consistency relation

$$r = 4(1 - n_s - n_t) + \frac{4n_t'}{n_t^2 - 2n_t'}\left(1 - n_s - \sqrt{2n_t' + (1 - n_s)^2 - n_t^2}\right)$$

- n_t can be positive in contrast with the unrenormalized result $n_t = -2\epsilon < 0$
- The running index $n_t' \equiv \dfrac{dn_t}{d\ln k}$ has to be measured to check the new consistency relation.
- The new consistency relation is compatible with a non-zero value of $r \approx 4(1 - n_s)$ even if the tensorial spectral index n_t is zero (in contrast with the standard result $r = -8n_t$).

Hopefully, these new predictions will be inside the range of measurements of the PLANCK satelite

However, we can already (partially) compare with observational data (WMAP 5-year)

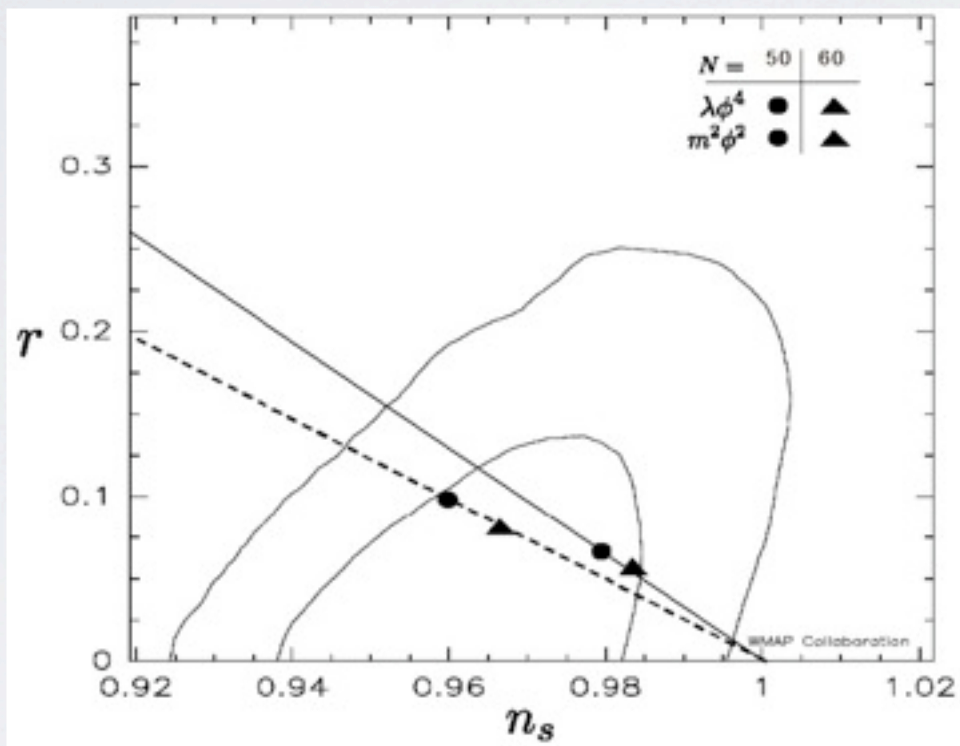
Let us take the representative monomial potential

$$V(\phi) = \lambda\phi^p$$

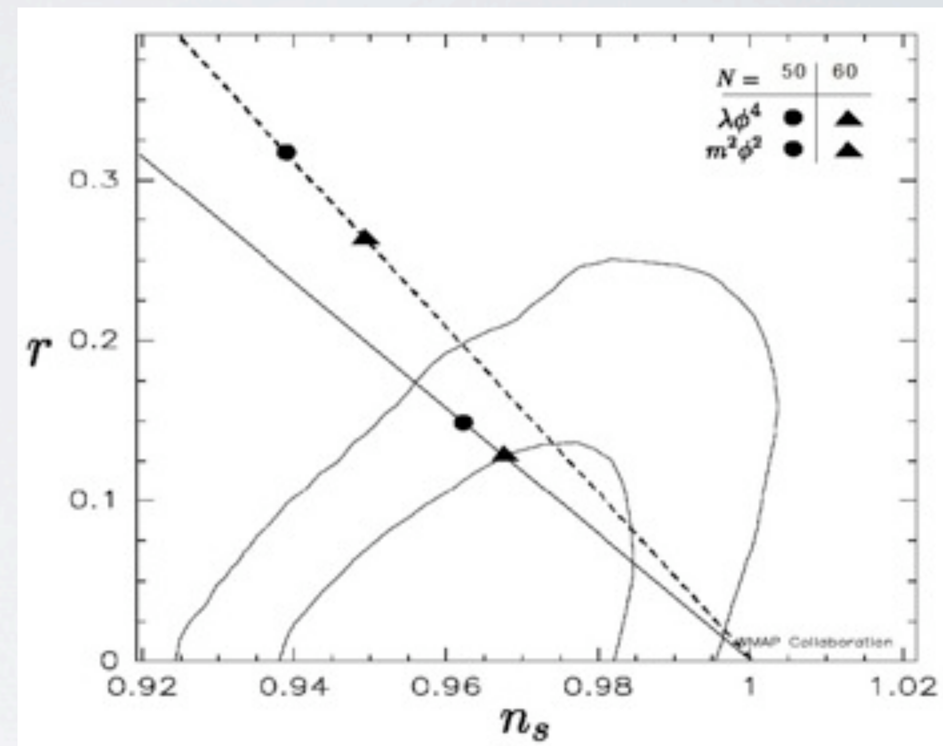
——— $V(\phi) = m^2\phi^2$

- - - - - - - $V(\phi) = \lambda\phi^4$

Renormalized



Unrenormalized



N is the number of e-folds (Hubble times) of inflation between t_k and the end of inflation ($N \in [50, 60]$)

$$r \equiv \frac{P_t(k)}{P_{\mathcal{R}}(k)} = 16\epsilon \frac{\epsilon}{(3\epsilon - \eta)}$$

Scalar spectral index $$n_s - 1 = -6\epsilon + 2\eta + \frac{(12\epsilon^2 - 8\epsilon\eta + \xi^2)}{3\epsilon - \eta}$$

Tensorial spectral index $$n_t = 2(\epsilon - \eta)$$

Running of the tensorial spectral index $$n_t' \equiv \frac{dn_t}{d\ln k} = 8\epsilon(\epsilon - \eta) + 2\xi^2$$

$$\xi^2 \equiv M_P^4 (V'V'''/V^2)$$